XXII.

STUDIES IN METROLOGY.

FIRST PAPER.

By William A. Rogers.

Presented May 9th, 1883.

It will be the aim of the present paper to present a critical study of certain standards of length, which have been compared either directly or indirectly with the original standards, which are now recognized throughout the civilized world as the ultimate and supreme authority in all matters relating to units of length; viz. with the "Imperial Yard" at London, and with the "Metre des Archives" at the International Bureau of Weights and Measures, situated at Breteuil, near Paris.

The second paper will contain a discussion of all the standards of length which have been constructed by the writer from the prototypes investigated in this paper. These prototypes are designated and defined as follows: —

I. *The Tresca Meter, having the designation T.*

The bar upon which the defining lines of this meter are traced is composed of pure copper, and has narrow strips of platinum inserted at each end. The bar has the X shape proposed by Professor Tresca, which allows the graduations to be placed nearly in the plane of the neutral axis without interfering materially with their examination under a microscope having considerable magnifying power.

The platinum surfaces are in the same plane with the surface of the copper, not only at each end, but throughout the entire length. These surfaces are fairly well adapted to receive sharply defined graduations; but this material is far inferior to platinum-iridium in this respect.

This bar was placed upon the tracing comparator of the Conservatoire des Arts et Metiers, on the morning of February 4, 1880. After remaining at a constant temperature for about forty hours, M. Gustave

Tresca, at two o'clock on the morning of February 6, transferred to it the Conservatory line meter No. 19, whose relation to the Metre des Archives had been previously determined with great exactness by Professor Tresca. There are three defining lines at each end, with an interval of 17.4 μ. The width of each line is about 5 μ. It appears from a large number of subsequent comparisons of the three meters defined by these lines, that they do not differ *inter se* by a measurable quantity.

Immediately after the transfer was completed, a direct comparison with meter No. 19 was made. Comparisons were continued during the next day and the next night.

The certificate which accompanied this meter states that it is 1.00 μ longer than the corrected value of meter No. 19 at 13°.70 C.

Employing in the reduction the coefficient of expansion for No. 19, .00000860860, we find, therefore, that

T is 118.94 μ *too long at* 13°.70 C.

II. *The Froment Meter, designated F.*

This meter was originally an end measure only. The metal is steel, but the end surfaces were not hardened. This meter has been compared indirectly with the Metre des Archives through a comparison with a meter belonging to the Observatory of Kazan, which, by a comparison with the meter of the Conservatory, had been found to be 2.431 μ too long. During a visit to the establishment of Messrs. Demoulins-Froment, I ascertained that M. Froment had a line meter upon steel, which was supposed to be the equivalent of the end meter. Having purchased the latter, I asked M. Diner, who has this class of work in charge, to transfer to its upper surface this line meter, with subdivisions into decimeters. Inasmuch as the upper part of the bar extends beyond the defining surfaces of the meter at each end, this transfer was fortunately feasible. The transfer by M. Diner was made with great care, and from a comparison which I was permitted to make with the comparator of M. Froment, I became convinced that the claim of M. Diner, that the limit of error did not exceed 2 μ, was clearly substantiated.

The defining surfaces of the end meter are parallel only in the vertical plane, which is perpendicular to the axis of the bar. It will appear from the discussion which will follow, that, if the measurements are made from contacts on either side of the vertical line passing through the centre of the bar, an error will be introduced which is

proportional in magnitude to the distance from this line. Its maximum value is about 8 μ.

In this discussion the end meter will be designated F_e, and the line meter F_l. We have, then,

$$F_e - 8.43\ \mu = \text{Metre des Archives.}$$
$$F_l = F_e \text{ nearly.}$$

Upon placing F_e upon my own comparator after my return to Cambridge, I was surprised to find that, when the terminal lines were in focus, the lines along the middle of the bar were barely visible under the microscope. I was the more surprised, as I had examined the bar with respect to this very point while at Paris. After becoming assured that the ways of my own comparator had no sensible flexure, by certain tests which will be described later in this paper, only one conclusion could be reached; viz. either that the microscopes employed in the examination at Paris had not sufficient power to enable the observer to detect the deviation of the ways of the comparator from a horizontal plane, or that the curvature of the ways was the same as the curvature of the bar.

Inasmuch as it was not possible on this account to determine exactly the relation of each decimeter of F_l to the entire meter, and because the softness of the metal prevented the repetition of exact end contacts, I decided to reconstruct this bar for my own use. Before this was done, however, the following operations were performed.

(*a*.) The terminal lines of F_l were transferred to T at 13°.70 C. nearly.

Two sets of graduations were made, one composed of five rather coarse lines, and the other composed of five lines having as nearly as possible the same characteristics as the Tresca defining lines. The former are designated $T^{b_1 \cdots 5}$, and the latter $T^{c_1 \cdots 5}$. Corresponding lines were traced at the middle point of T upon the surface of copper. At the same time, a provisional yard was laid off upon T, employing one defining line of the meter as the defining line of the yard at that end.

(*b*.) The relations between F_l and $T^{a_1 \cdots 5}$ and $T^{b_1 \cdots 5}$ were carefully determined.

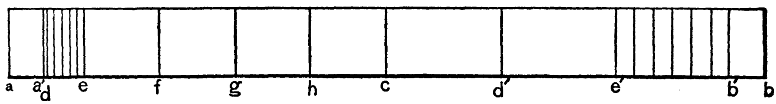
(*c*.) The relation of F_e to $T^{a_1 \cdots 5}$, and also to three other meters to be described hereafter, was determined.

After this work was done, hardened steel plugs were inserted at the ends of F_e, and the surfaces were made parallel. In order to accomplish this, it was necessary to grind off a trifle more than the 8.4 μ,

by which amount F_e was longer than the meter of the Archives. The amount by which the length was actually diminished will appear in the discussion which follows.

The upper surface of this bar was made nearly a plane surface when supported at its neutral points. The amount of metal removed at the middle point was about .012 inch. Tempered steel plugs were then inserted with a close fit in the upper surface of the bar, according to a scheme shown in the following sketch: —

Diagram Illustrating Subdivisions of the Yard and Meter.



METRIC.

a b = 1 Meter.
a c = c b = ½ Meter.
a e = e f = f g = g h = h c = 1 Decimeter.
a d = d e = 5 Centimeters.
d e subdivided to Centimeters.

ENGLISH.

a′ b′ = 1 Yard.
a′ c = c b′ = ½ Yard.
c d′ = d′ e′ = e′ b′ = 6 Inches.
e′ b′ subdivided to Inches.

By means of a device suggested by Mr. George F. Ballou, who prepared the bar, the surfaces of these plugs were brought into the same focal plane after they had received the final polish. At this point it will be proper to explain the method by which the general surface of graduated bars is made plane.

As will be hereafter shown, a plate to which a microscope is attached slides freely upon the ways of the comparator. The flexure of these ways is controlled by means of flexure screws beneath. It is evident that, if the objective of a microscope of high power remains in good focus at every point along a polished metal surface lying parallel with a plane passing through the points of contact at the two ends of the ways, the movement of the microscope plate along the ways defines a reference plane with which any other plane can be compared. The following tests have been employed in the definition of this reference plane.

(1.) Mr. George B. Clark, of the firm of Alvan Clark & Sons, prepared for me a steel bar, one meter in length, which has one optically plane surface, *when the bar is supported at its neutral points.* Placing the bar thus supported upon the comparator, and adjusting it so that surfaces at the two ends are in a focal plane parallel with that defined by the microscope plate in corresponding positions, it has been found that the objective remains in focus at every point along the surface of this bar.

(2.) A similar test has been applied with a surface plate $42 \times 1\frac{1}{2} \times 6$ inches, prepared by Professor Morris of Cornell University. I have two of these surface plates. One was prepared in the usual way, but the other was not touched with a tool after it left the planer. I think it would be difficult to find a better specimen of planing. Although each plate weighs about twenty-five pounds, one will lift the other by the contact of their surfaces.

(3.) If a shallow dish of mercury is placed upon the comparator, the surface of the mercury forms a reference plane with which the plane described by the microscope plate can be compared. Under a half-inch objective supplied with a Tolles opaque illuminator, one can focus upon the minute globules of mercury with considerable sharpness.

Having proved that the microscope plate describes a true plane during its movement along the ways, the bar whose surface is to be prepared is placed upon the comparator, *supported at its neutral points*, and the surface is worked down with soft stone till every part remains sharply in focus under the objective.

Experience has shown that the surface of every bar prepared in this way falls into a plane surface within a short time after it is placed upon its supports.

(4.) Recently a fourth method of testing the deviation of the plane described by the microscope carriage from a true horizontal plane has been applied with good success. It may be described as "the method by two microscopes." A flat bar, about sixty centimeters in length, was attached firmly to the carriage in a direction parallel with the ways. At each end, the arm which supports the microscope was fastened with an adjustable motion in every direction. The two microscopes when placed in position are one half-meter apart, and equidistant from the centre of the carriage.

The Clark reference bar having been placed in position upon the bed of the comparator, the defining lines of each end were adjusted for focus under the right-hand microscope. The flexure screws beneath the comparator were then moved till the defining line at the middle point was in sharp focus. It is now found that every part of the surface of the reference plate remains in good focus. But it does not follow that either the plane described by the microscope carriage or that of the reference bar are real planes. It can only be said that they are parallel. We now adjust the left-hand microscope upon the reference bar at the left end. Now, if the microscope carriage describes a true plane in passing from the middle point to the right end of the comparator, the defining line near the middle point of the bar at which

the other microscope had been adjusted for focus *will remain in focus under the left-hand microscope.*

This operation will test the movement of the carriage over only one half of the distance moved, viz. from the middle to the right end. But by adjusting the left microscope upon the right end of the bar, and moving the carriage from right to left, the remaining half can be tested in a similar manner. It is found that only two or three trials need to be made in the adjustment of the bed-plate by means of the flexure screws in order that the defining lines at each end of the reference bar shall remain in sharp focus under both microscopes. Of course, the lines at the middle of the bar will now remain in focus only when they are in the same plane as the terminal lines. In the case of the Clark reference bar this condition is fulfilled with great exactness. It will be noticed that this method does not require that the surface of the reference bar shall be a true plane. Any two points may be chosen, but these points must remain in the same plane during the entire operation of testing.

Since this method has been adopted, much better results have been obtained than it was possible to obtain before.

(5.) A fifth method avoids dependence upon the variation of focus as the test of a plane surface. In this method the microscope is attached to the carriage in a horizontal position, the line of sight being a little above the surface of a bar which is supposed to be a plane surface, and with which the plane described by the microscope carriage is to be compared.

If a small piece of metal having one plane surface is placed at different positions upon the surface of the reference bar, a given point upon the front face, formed by the intersection of a perpendicular and a horizontal defining line, will remain in a plane parallel with the surface of the bar. This test-piece is first placed at one end, and the defining horizontal line is brought into coincidence with the defining line of the microscope micrometer. The carriage is then moved to the other end, and the test-piece is moved upon the surface of the bar till the cross lines are brought into the field of the microscope. The coincidence with the micrometer line is then made by the vertical adjustment of the bar. The test-piece is now placed at the centre of the reference bar, and, if the coincidence of the defining lines is still maintained, the surface described by the carriage is parallel with the surface of the bar. The deflection of the ways at any point can thus be measured by means of the micrometer screw of the microscope.

This method of preparing the surfaces of bars for graduation obvi-

ates the inconvenience of placing the defining lines at the bottom of wells in order to reach the plane passing through the neutral axis, and it enables one to place the terminal lines and the lines marking the subdivisions in the same plane.

The steel bar prepared in this way is designated R_1. Three sets of graduations are traced upon the polished surfaces of the steel plugs. They are designated as follows: —

$$R_1^{a_{12}}, \quad R_1^{b_{123}}, \quad R_1^{c_{12345}}.$$

Lines $R_1^{a_1}$ define the meter and the yard provisionally at 62°.0 Fahr. The corrections required for these defining lines having been ascertained from a sufficient number of observations, they were applied in tracing the remaining graduations. The lines of the group R_1^c vary in width between 0.6 μ and 5 μ. They are therefore well adapted to furnish the data required in order to determine the effect of the size of the lines upon the error of focus under the objective.

III. *The Bronze Yard and Meter, designated* R_2.

The material of this standard is an alloy known as Bailey's metal. It consists of 16 parts of copper, $2\frac{1}{2}$ parts of tin, and 1 part of zinc. It is identical in composition with the Imperial Yard, and differs from it only in having the graduations upon platinum-iridium plugs, which have their polished surfaces in the plane of one surface of the bar when it is supported at its neutral points, instead of having them upon plugs of gold situated at the bottom of wells sunk to the plane of the neutral axis.

This standard forms the basis of the units which the writer undertook to obtain for Messrs. Pratt and Whitney, of Hartford, Connecticut, and upon which their system of gauges depends.

Both the yard and the meter are nearly standard at 62° Fahr. The defining lines of the meter were obtained by transfer from the Tresca meter. The defining lines of the yard were obtained indirectly from the Imperial Yard, in the following manner.

Early in 1880, Mr. Chaney, Warden of the Standards, did me the great service to compare a yard traced upon the surface of a steel bar with the Imperial Yard. According to the observations of Mr. Chaney, the yard traced upon this bar was .000316 of an inch too short at 62° Fahr. Unfortunately, this bar had been nickel-plated, and the graduations were traced upon the nickel surface. Subsequent investigations revealed the fact that the expansion and contraction of this bar were

very irregular, and that the normal action under a given change of temperature could be obtained only by maintaining a constant temperature for about twelve hours. Fortunately, a portion of the comparisons with the Imperial Yard were made under the required conditions, at the nearly constant temperature of 57° Fahr. When it became apparent that the transfer at 62° Fahr. from the steel bar to a bronze bar identical in form and construction with R_2 could not be made with the required exactness, it was decided to make the required transfers under conditions as nearly as possible the same as those under which the comparisons had been made at London. After maintaining the nearly constant temperature of 57° for about twenty-four hours, this transfer was made to a bronze bar designated R_0. After applying the reduction from 57° to 62°, Professor J. E. Hilgard, now Superintendent of the U. S. Coast Survey, having kindly offered to compare this yard with the standard yard known as "Bronze 11," which bears a known relation to the Imperial Yard, bar R_0 was sent to Washington for this comparison. According to the report of Professor Hilgard, this yard was found to be 24 millionths of an inch longer than "Bronze 11," or 64 millionths of an inch shorter than the Imperial Yard, adopting the relation

"Bronze 11" + .000088 in. = Imperial Yard.

It was now possible to make the transfer from R_0 to R_2 with but little regard to the question of temperature, and with a known relation to the Imperial Yard. This transfer was made, January 13, 1881. The yard R_2 is subdivided into feet, and the first foot is further subdivided into inches. At one end a single line defines the limit of both the yard and the meter. The other defining line of the meter extends beyond the corresponding defining line of the yard 3.37 inches. For convenience in measuring this space, the yard is extended to 39 inches, with inch subdivisions. Subsequently the system of graduations upon R_1 were transferred to the brass surface of this bar, being traced near the edge. This set of graduations is designated R_{2e}.

IV. *The Brass Yard and Meter designated C. S.*

This standard was prepared by the U. S. Coast Survey many years ago, and presented to the Stevens Institute at Hoboken, N. J. By the kindness of President Morton I was permitted, in January, 1880, to take this standard to London in order to obtain a comparison of

the yard with the Imperial Yard. This comparison was made with great care by Mr. Chaney, and, at his request, by the writer also. It was then sent to Breteuil for comparison of the meter with the International Meter. The bar was received from Paris, February 4, 1883. According to the report of Dr. Pernet, this meter is $310\,\mu$ *shorter than the Metre des Archives at* 0° C. Inasmuch as the comparisons with the prototype were made near 1°, 7°, and 12° Centigrade, with the greatest exactness, the data are at hand for the determination of the length of this metre at 16°.67 C., or 62° Fahr., but they have not yet been received. It is, however, quite as well that the present investigation should have been made independently, and without any knowledge of the results obtained at Breteuil.

The graduations are traced upon plugs of silver inserted at the bottom of wells sunk to the line of the neutral axis. The defining lines of the meter present sharply defined edges, but one of the defining lines of the yard has a somewhat broken appearance, being broader at some points than at others. Two lines about 3 mm. apart are drawn at right angles to the defining lines. It appears, from repeated examinations, that the defining lines, both for the meter and the yard, are sensibly parallel. The comparisons made by the writer have been made at points about midway between the horizontal lines.

V. *The Glass Yards and Meters designated G_1 and G_2.*

These bars were made by Chance and Sons in 1870, for the Standards Department, London. G_1 was presented to the writer by Mr. Chaney, under the authority of the Board of Trade, in 1880. G_2 is the property of the Standards Department. It was intrusted to the writer for the purpose of graduation.

If one may judge from the character of the lines ruled with the diamond, these bars are far from homogeneous throughout their whole extent. With the same pressure upon the diamond, some of the lines are slightly broader than others, and while nearly all of the lines are of good quality, a few of them present a decided granular structure.

In the transfer of the graduations from one standard to another having a different composition, I have found it advisable to first make a provisional transfer, with an assumed relative coefficient of expansion between the two bars, and with an assumed series of corrections to the prototype on account of the horizontal curvature of the ways.

The bars are then placed upon the universal comparator, and the relations between the prototype and the standard constructed from it

are determined by a sufficient number of observations, at a wide range of temperature. This operation generally requires two or three weeks. With the corrections thus determined, a second series of graduations are laid off, the lines being in the same field of the microscope as the first ones. One can now proceed nearly independently of temperature, by comparison with the first set of graduations.

Even when the correct length is obtained, it is often found necessary to make a third, and even a fourth set, in order to reduce the errors of the subdivisions to sufficiently narrow limits. According to my present arrangement, after a transfer has been made, all the subdivisions, both of the meter and the yard, can be made without disturbing the position of the two bars. But with the utmost care I have not found it possible to make the subdivisions exactly equal. In fact, one must be content to admit errors of small magnitude as long as the conditions of transfer vary in the slightest degree from the conditions under which the required corrections were obtained. The corrections due to curvature give great trouble, and require the most careful determination. With the universal comparator, this difficulty is obviated; but on account of the better illumination obtained in the comparing-room at the Observatory, all transfers have thus far been made upon the comparator first constructed.

The provisional set of graduations upon G_1 and G_2 were laid off from $R_1^{a_2}$, on January 7, 1883. They consist of bands of three lines, separated by an interval of $50\,\mu$. They are designated $G_1^{a_{123}}$ and $G_2^{a_{123}}$.

The relations between R_1, G_1, and G_2 were then determined from observations extending from January 8 to January 17.

In July of the present year, G_1 and G_2 were placed upon the comparator for the transfer of the graduations $G_2^{a_2}$ to G_1, giving a set of graduations designated $G_1^{b_{12345}}$. After the comparison of the latter with $G_1^{a_2}$, and with the series of corrections thus obtained, the bars were placed in reversed positions, and lines $G_1^{b_1}$ were made the basis of a new transfer to G_2, giving $G_2^{b_{123}}$.

Graduations $G_2^{b_2}$ having been compared with each other and with $G_2^{a_2}$, the following sets were laid off, viz.: —

$G_2^{c_{123}}$ with lines one half the width of $G_2^{b_{123}}$.

$G_2^{d_2}$ with lines having about the same width as $G_2^{b_{123}}$.

$G_2^{e_2}$ with exceedingly fine lines.

The distance between the lines in the final sets both of G_1 and G_2 is $50\,\mu$.

VI. *The Whitworth Steel Yard, designated W.*

This yard is an end-measure standard. It was purchased of Sir Joseph Whitworth & Co., in 1880, at the cost of about $70. No statement of the correction required for the length of the yard at 62° accompanied it. One may infer, therefore, that it was considered standard at that temperature. The terminal surfaces which define the yard are one half-inch in diameter. All the observations which have been made indicate that these surfaces are exactly parallel. The steel has been tempered at each end, the temper extending about one inch.

This bar has been included in the present list of prototypes on account of the relation which it may be supposed to bear to the system of gauges universally adopted in Great Britain. The original from which the Whitworth steel standards were constructed seems to have been a bronze bar made by Troughton and Sims at the same time the national standards were made by these distinguished mechanicians. It does not appear that this yard was included among those compared by Sheepshanks. Probably, however, its relation to them was determined by Mr. Sims. I have been unable to find any record of a direct comparison with the Imperial Yard, but without question the correspondence must be very close. The transfer of this yard to steel involves many considerations which do not enter into a comparison of standards having the same composition as the Imperial Yard. The great service which Whitworth has rendered to the science of metrology would lead us to expect that the steel yard which is the result of his researches truly represents the bronze yard when converted into its equivalent upon steel at 62°. Nor is this expectation disappointed. A certain allowance must always be made for the different conditions under which independent investigations are made. It will not do to say that the error is all on one side. But admitting that the Whitworth steel standard really differs from the steel standard yard R_1 by the amount indicated by this discussion, it yet remains true that this difference will disappear from the exact subdivisions of the yard with the units which are required in exact mechanical operations, viz. the inch and its multiples.

The important data with respect to the prototypes described above are presented in the following table.

DIMENSIONS.

Bar.	Weight.	Length.	Width.	Depth.
T	3.1 lb.	101 cm.	2.0 cm.	2.0 cm.
$C. S.$	7.1 "	102 "	2.5 "	1.4 "
$R_1 = F$	7.6 "	101 "	1.4 "	3.0 "
R_2	12.8 "	102 "	2.5 "	2.5 "
G_1	12.5 "	102 "	4.0 "	4.0 "
G_2	12.5 "	102 "	4.0 "	4.0 "
W	9.3 "	91 "	2.5 "	2.5 "

WIDTH OF DEFINING LINES.

$T^{a_{123}} = 5\,\mu$	$G_1^{a_{123}} = 5.0\,\mu$	$R_1^{a_1} = 7.0\,\mu$
$T^{b_1 \cdots 5} = 7\,\mu$	$G_1^{b_1 \cdots 5} = 3.0\,\mu$	$R_1^{a_2} = 4.0\,\mu$
$T^{c_1 \cdots 3} = 4\,\mu$	$G_2^{a_{123}} = 4.0\,\mu$	$R_1^{b_{123}} = 3.1\,\mu$
$C.S.$, meter $= 18\,\mu$	$G_2^{b_{123}} = 3.0\,\mu$	$R_1^{c_1} = 0.6\,\mu$
$C.S.$, yard $= 7\,\mu$	$G_2^{c_{123}} = 1.5\,\mu$	$R_1^{c_2} = 1.2\,\mu$
$C.S.$, max. $= 10\,\mu$	$G_2^{d_{123}} = 3.0\,\mu$	$R_1^{c_3} = 2.0\,\mu$
$C.S.$, min. $= 6\,\mu$	$G_2^{e_{123}} = 0.5\,\mu$	$R_1^{c_4} = 7.7\,\mu$
$R_2^{a_{123}} = 5\,\mu$		$R_1^{c_5} = 3.4\,\mu$

The discussion of these prototypes will proceed in the following order: —

(*a*) Description of comparators.
(*b*) Description of comparing-rooms.
(*c*) Discussion of thermometers.
(*d*) Description of microscopes. Values of the micrometer screws.
(*e*) Investigation of the time required for a given change of temperature to produce its normal effect upon bars having different masses.

(*f*) 1. Report of Professor Hilgard upon the comparison of $R_2^{a_2}$ with "Bronze 11," made by Assistant Edwin Smith.
2. Comparison of $R_2^{a_2}$ with "Bronze 11," at Washington, with Comparator No. 1.
3. Comparison of T^{a_2} with Meter "No. 49," at Washington.

(*g*) Preliminary comparisons of standards at Cambridge with Comparator No. 1, and with the Universal Comparator.
(*h*) Investigation of coefficients of expansion.
(*i*) Final comparisons of prototype standards.
(*j*) Probable error of observations.
(*k*) Tabular values of errors of subdivision.
(*l*) Investigation of the equation between the Imperial Yard and the Metre des Archives.

Comparator No. 1.

This comparator is of the most simple form. It consists of a bed-plate 60 inches in length, 14 inches in breadth, and is ribbed to the depth of 2½ inches. V-shaped ways extend the entire length. A carriage 8 × 6 inches is carried along these ways, either by hand movement or by a rack and free pinion. The rack, which is placed midway between the ways, extends the whole length of the bed-plate, and the shaft which carries the pinion is placed at the centre of the carriage. On each side of the centre, two independent platforms are mounted with vertical adjustments, to which the microscopes are attached. There is, besides, sufficient room at one end of the carriage for the tracing apparatus, which may thus be made to maintain a fixed relation to either microscope.

Gibbed grooves extend the whole length of the bed-plate, to which are fitted two stop-plates, 4 × 5 inches, which can be clamped firmly to the bed at any point. Oval-shaped tempered steel stops are inserted in the end foces of these plates, and corresponding flat surfaces of steel are inserted at each end of the microscope carriage.

The flexure of the bed-plate is taken up by means of two screws, which are attached to an independent plate beneath and near the middle point.

The V-shaped ways have a decided horizontal curvature, which gave great trouble until the form of the curve had been investigated. Several independent investigations of the effect of this curvature have been made, both for the yard and the meter, with the following results.

	For one-inch objective.	
	Correction to be applied to a measured yard for each inch of distance from the centre of the ways.	Correction to be applied to a measured meter for each centimeter of distance from the centre of the ways.
Mean of values derived from observations previous to Jan. 1, 1881	div. 10.50	div. 4.74
Values obtained at Washington .	10.43	4.57
Values derived from equations of condition subsequent to 1881. .	10.46	4.58
Values derived from direct measures subsequent to 1881 . . .	10.43	4.26
	10.45	4.54

The general principle of this form of construction, and the methods of comparison employed, will more clearly appear from the drawings and description of the Universal Comparator which follow.

The Rogers–Bond Universal Comparator.

This comparator is the outgrowth of experience with the apparatus first constructed. In 1880, through the kind offices of Professor J. E. Denton, of Stevens Institute, Mr. George M. Bond, then a member of the Senior Class, came to Cambridge and made the sketches, from which he afterwards executed the full drawings. Mr. Bond was intrusted with the entire arrangement of all details, and to his judicious application of sound mechanical principles in the form of construction is due in a large measure the successful working of the apparatus.

Soon after this, Messrs. Pratt and Whitney, of Hartford, Conn., undertook the construction of two machines from these specifications, one of which was to be used as an auxiliary in the investigation and construction of the system of gauges which they have established. These comparators were completed in May, 1881; but it was not until the September following that the machine constructed for the writer was completely mounted at Cambridge.

In designing this apparatus, the following requirements were kept steadily in view: —

(1.) The complete separation of the standards to be compared from the frame-work to which the microscopes are attached.

(2.) The best method of securing an invariable reference plane,

defined by the motion of the microscope along the ways upon which it rests.

(3.) Mechanism for the adjustment of the bars under the microscopes in the shortest possible time, by a combined quick and slow motion, which should not require any disturbance of the standards after they are placed in position upon their neutral points of support. The three quick and slow movements required are:

(*a.*) A motion of translation, longitudinally.

(*b.*) A motion at right angles to the motion of translation.

(*c.*) A vertical motion.

(4.) Mechanism which will allow comparisons of line-measures by as many independent methods as possible.

(5.) Mechanism for the convenient comparison of line-measures with end-measures.

(6.) Mechanism which will allow an easy comparison of the subdivisions of any unit, without regard to their magnitude.

Figures 2 and 3 present perspective views of the apparatus, and show the general relations of all the parts to each other. The plan of the comparator is shown in Fig. 4. The front elevation is shown in Fig. 1, in which the part above the line A B is lifted, in order that it may be seen clear of the tables. The form of the micrometer of the microscope is shown in Fig. 6. The general form of the microscope carriage, with microscope in position, is shown in Fig. 7. An end view of the carriage G is shown in Fig. 5.

The first condition named is fulfilled by mounting the bed-plate L L, Fig. 1, upon brick piers having granite cap-stones. The bed-plate of the carriage G, which supports the tables S, rests upon independent piers. Both of these sets of piers are disconnected from the floor of the comparing-room.

The second condition is fulfilled by the use of the hollow steel cylinders X. The microscope plate, K, moves freely upon these cylindrical ways. It can also be moved by a rack and free pinion. The rack is behind the cylinder X, but the head, R, of the pinion is shown. The flexure of the cylinders is taken up by the levers P, T, which are not shown in their proper position. They should be nearer the centre. The required pressure is supplied by weights attached to cords fastened to the ends of the lever P.

It is due to the excellent workmanship of the Pratt and Whitney Company that these ways have no appreciable horizontal curvature, while the very slight deflection due to flexure is readily taken up by the flexure levers. An experience of nearly two years has shown that

the reference plane described by the movement of the microscope carriage is nearly invariable.

The third requirement is met by the following arrangement.

The standards to be compared are placed upon the table S^2, Fig. 1. The whole framework to which this table is attached can be raised or lowered by the hand-wheel C. It can also be moved forward and backward, upon the ways shown in Figs. 3 and 4, by means of bevelled gears driven by the hand-wheels beneath the bed-plate. These coarse movements are designed to facilitate placing the standards to be measured approximately in the proper position in relation to the microscopes M, M^1.

The carriage S is moved rapidly upon the V-shaped ways at the upper surface of E, by a rack and free pinion movement. The rack is shown at the left of the figure, while the handle of the pinion is shown at D.

The standards having been placed in such a position upon the table S that the defining lines at one end shall be nearly over the fulcrum below S^3, the coarse motion in a direction at right angles to the motion of the carriage S is obtained by the gibbed slides, S^8.

The slow movement of the carriage is obtained as follows: —

(1.) In translation, by the lever whose handle is shown at S^5.

(2.) At right angles to the motion of translation, by the lever S^6.

(3.) Vertically, by means of the screw S^3 shown in the left-hand carriage.

In practice it is found that these levers give a more delicate adjustment than can be obtained with a screw movement, and quite as good as can be obtained with the micrometer screw of the microscope. It will be observed that the position of the standards with reference to the table S is not disturbed by any of these adjustments. The time required in order to bring the terminal lines of a yard or meter into proper position, adjustment, and focus under the microscopes, is about forty seconds.

Under the fourth requirement the following methods of comparison are described.

(*a.*) *Comparison by means of two Fixed Microscopes.*

The method of procedure will be illustrated by a description in detail of the comparison of the standards of which the results are given on the following pages. The following are the various steps of the operation: —

THE ROGERS-BOND COMPARATO

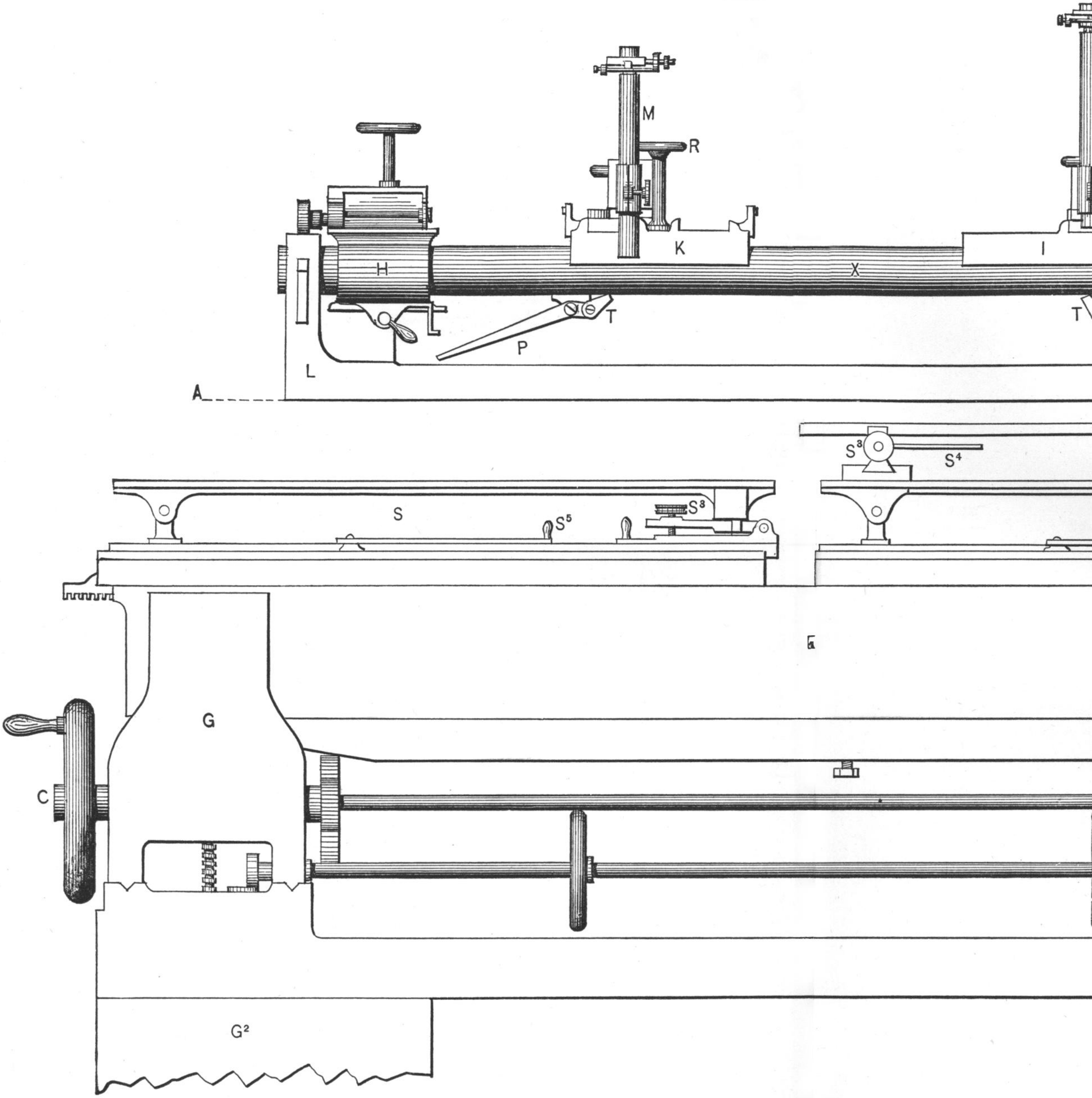


FIG. 1. FRONT ELEVATION.

Part above line A B *lifted to show clear of tables.*

HE ROGERS-BOND COMPARATOR.

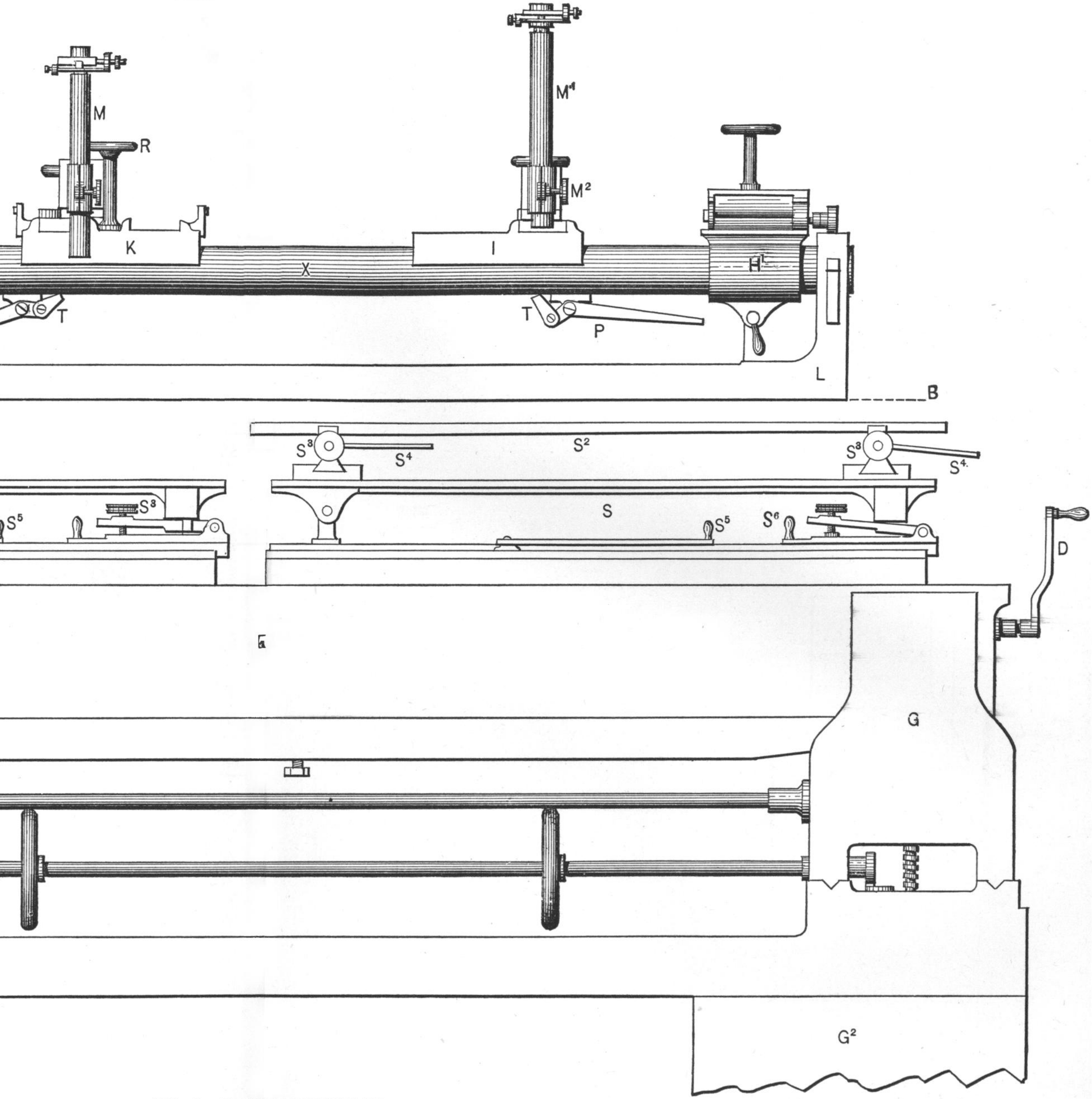


FIG. 1. FRONT ELEVATION.

Part above line **A B** *lifted to show clear of tables.*

THE ROGERS-BOND COMPARATOR.

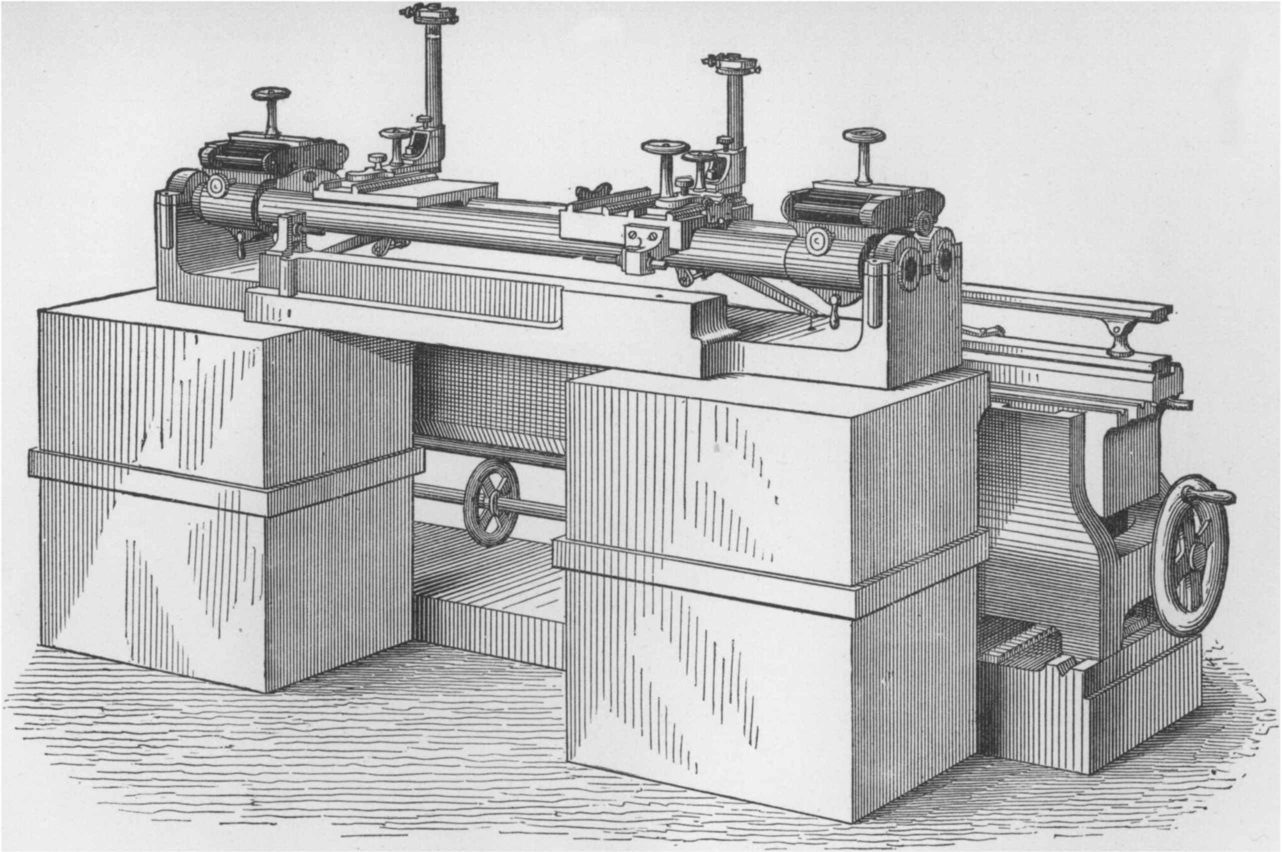


FIG. 2. FRONT VIEW, TABLES FOR STANDARDS, ETC.

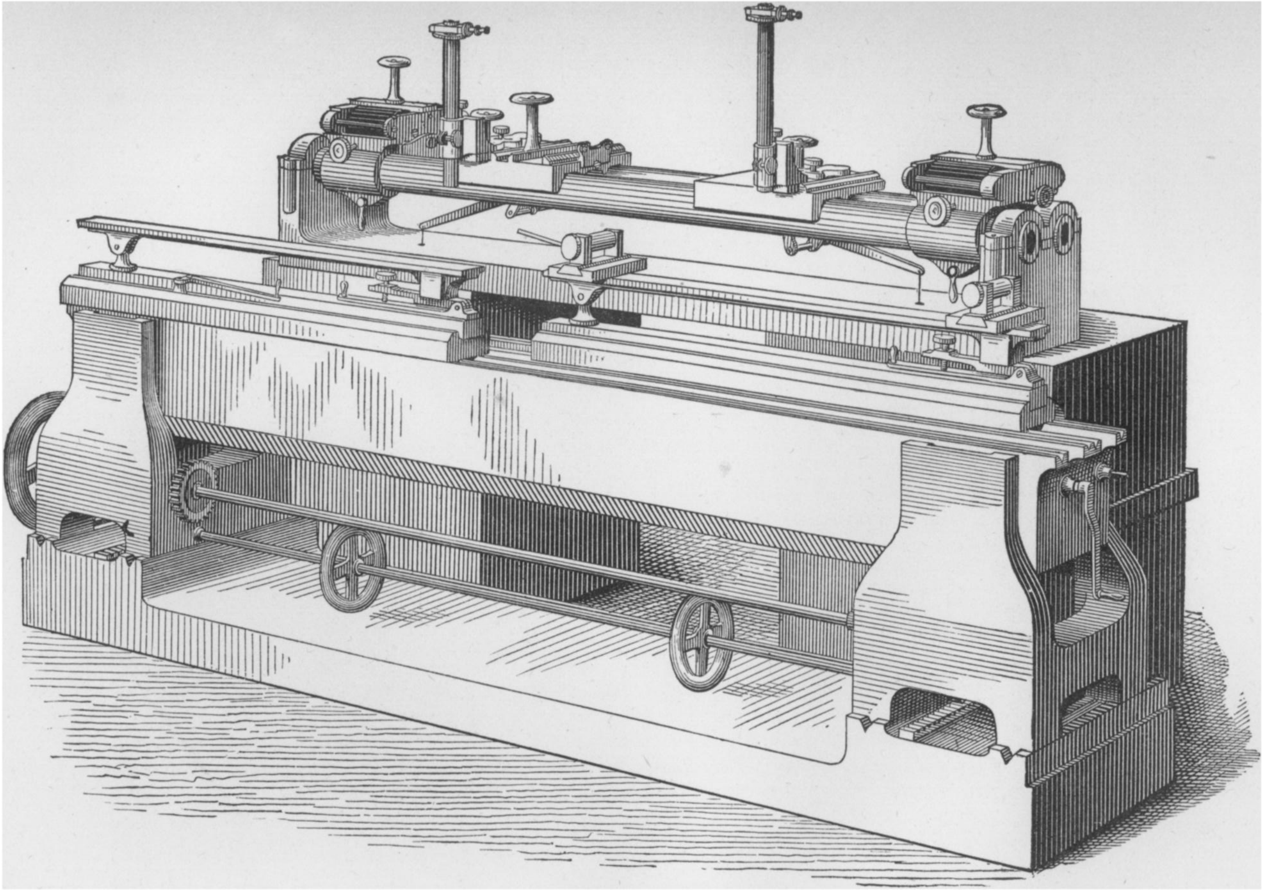


FIG. 3. REAR VIEW.

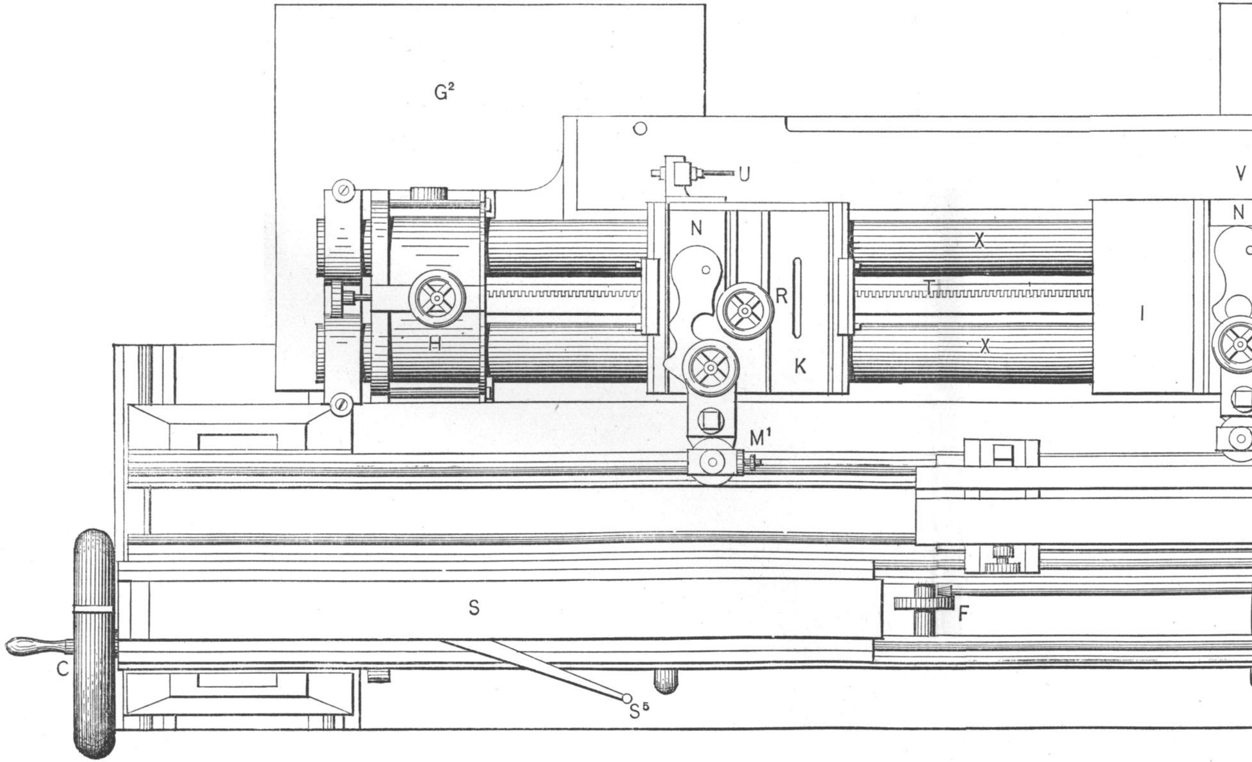


FIG. 4. PLAN OF COMPARATOR.

HE ROGERS-BOND COMPARATOR.

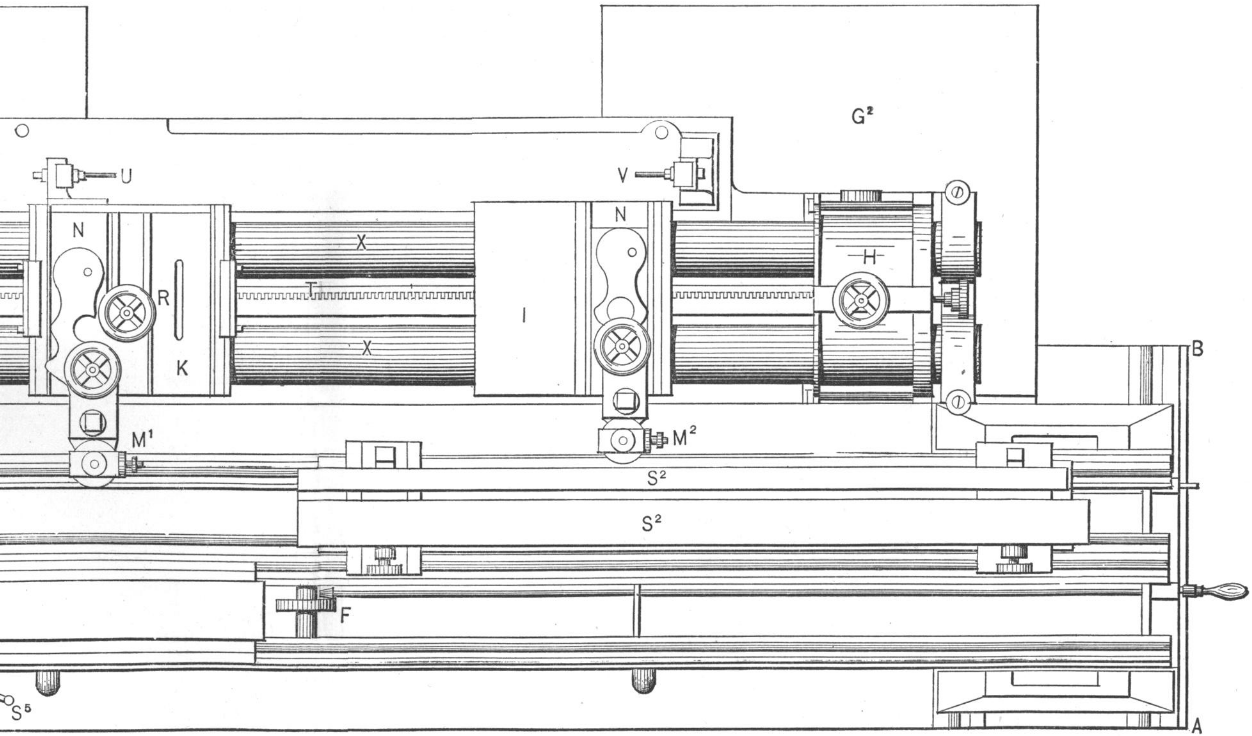


FIG. 4. PLAN OF COMPARATOR.

THE ROGERS-BOND COMPARATOR.

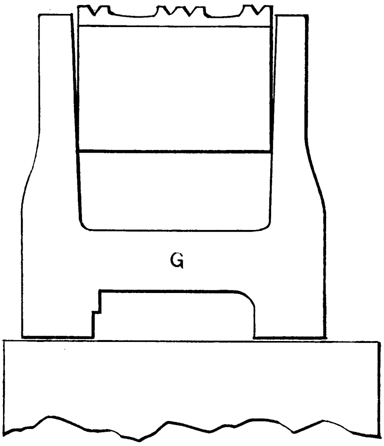


FIG. 5. END OF CARRIAGE G IN FIG. 1.

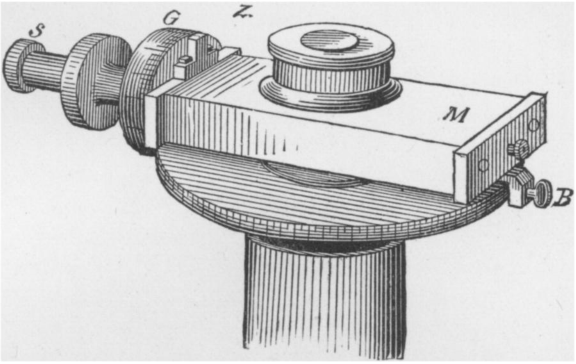


FIG. 6. THE MICROMETER.

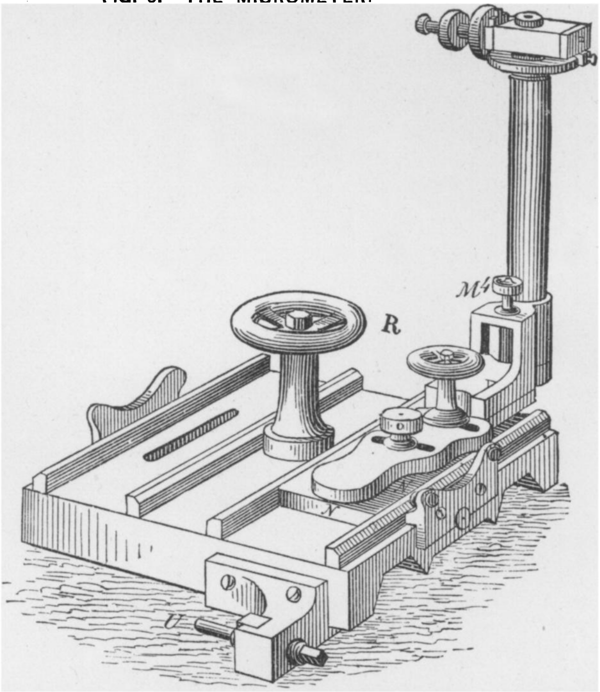


FIG. 7. THE MICROSCOPE CARRIAGE.

(1.) Adjustment of the standards T, $C. S.$, R_1, and R_2, upon supports at their neutral points, upon the table S^2. The defining lines at one end were placed nearly over the fulcrum at the left end of the carriage. T, having the least mass, is placed in front.

(2.) Permanent adjustment of the microscope plate I at a convenient distance from the end of the frame L.

(3.) Rapid movement of the carriage S by the handle D, until the right defining line of the meter T is brought nearly under the microscope M^1.

(4.) Movement of the plate K by the hand-wheel R, until the defining lines at the opposite end of T are brought into the field of the microscope M.

(5.) Adjustment of the left-hand defining line under the micrometer of the microscope M, by the horizontal movement of the slide S^3, and by the vertical movement S^4, and finally by the movement of translation through the lever S^5. The first two of these movements are made with sufficient exactness by the coarse adjustment. The defining line of the standard having been brought into coincidence with the fixed line of the micrometer of M by the fine adjustment S^5, no further change is made in the positions of K and I.

(6.) The adjustment of the right defining line by the motion of the micrometer of M^1, by the lever S^6, and by the vertical screw S^3. These adjustments will disturb the adjustment under M only in a slight degree. Usually, however, a second series of adjustments is necessary.

(7). The length of T having thus been transferred to the microscopes M, M^1, the standard $C. S.$ is then brought into position by the slides S^8, and the adjustments described above are repeated. The left defining line being coincident with the fixed wire of M, the difference in the length of the two standards is measured by means of the micrometer of M^1. An increasing reading of the micrometer-head corresponds to an increase of length.

(8.) Comparison of the meters R_1 and R_2 in a similar manner.

(9.) Adjustment of the defining line of the yard of $C. S.$ under M^1, by a rapid movement of the carriage S through the hand-lever D.

(10.) Movement of the plate K till the left defining line of the yard falls under the microscope M.

(11.) Repetition of the various adjustments and measures described above.

The average time required to make these adjustments, and to take four readings of M^1 for the defining lines T^{a_1}, T^{b_1}, for each edge of

C. S., for $R_1^{a_2}$, $R_1^{b_2}$, and $R_2^{a_2}$ of the meter, and the corresponding defining lines of the yard, is nineteen minutes. The average time of a single comparison of two units is therefore somewhat less than two minutes.

(b.) Comparison by means of two Microscopes attached to the same Movable Carriage.

The steps are as follows: —

(1.) The standards are placed in position upou the table S, one near the middle, and the other at a given distance in front of it.

(2.) The microscope K occupying a position near the middle of the comparator, the defining line at the left end of the bar is brought into focus of the microscope M by the screw M^4, Fig. 7. The carriage is then moved by the handle D till the defining line at the right end of either one of the standards is in the field of the same microscope. The adjustment for parallelism with the cylindrical ways is now made with the lever S^6 and for focus with the screw S^8.

(3.) Both microscopes are now placed upon the plate K. By means of the adjustments shown in Fig. 7, microscope M is adjusted upon the left-hand defining line of one bar, and M^1 is adjusted upon the corresponding defining line of the bar in front of it.

(4.) By a motion in translation through D, the right defining line of the first bar is brought into the field of M, and adjusted for coincidence with the fixed micrometer wire by the lever S^5.

(5.) The difference in the length of the bars is then measured with the micrometer of M_1.

(6.) The bar in front is now placed at an equal distance back of the first bar, and the operation described above is repeated.

(7.) The mean of the two results will be the difference in the length of the standards, free from the effect of the curvature of the ways along which the carriage moves. The same result will be obtained if the microscope carriage is moved along the cylindrical ways, the bars remaining stationary during each independent operation; but the curvature eliminated will in this case be that which belongs to the cylindrical ways.

(c.) Comparison by means of Stops and one Microscope.

The stops H H^1 move freely upon the cylinders X X, but they are capable of being clamped to them with great firmness by the levers shown near L. At one end they terminate in an oval projection of tempered steel, which is hidden behind the cylinder X, in Fig. 1.

It is nearly in line with the centre of gravity of K, which has a corresponding flat surface of tempered steel at each end.

These stops receive a slow motion by means of a screw, which is shown near the pillow blocks in Fig. 3.

(1.) The stops H H^1 are set approximately, e. g. 1 meter apart plus the length of K, and are securely clamped.

(2.) The plate K is brought into contact with the left-hand stop.

(3.) The standard to be compared is placed upon S, and the carriage is moved by the arm D till the left-hand defining line is adjusted under M.

(4.) The plate K is then brought into contact with the right-hand stop, and the other defining line is brought into position and into focus under M by the adjustments already described.

(5.) These adjustments having been completed, K is brought into contact, first with H, and then with H^1, and the micrometer is read for each contact.

(6.) These operations are repeated for each standard compared. Since each one has been compared with the invariable distance between the stops, the data are thereby furnished for the comparison with each other. In practice, the standard with which other standards are to be compared is always compared first with the distance between the stops, the record being written at the left of the page. The following example will illustrate the form of record. *R* signifies contact with the right stop, and *L* with the left stop.

Bar *T.*				Bar *C. S.*				$(R'-L')-(R-L)$
L		R	$R-L$	L'		R'	$R'-L'$	
rev.	div.	div.	div.	rev.	div.	div.	div.	div.
4	16.2	14.6	—1.6	4	19.7	19.9	+0.2	+1.8

Under these definitions, the second standard is longer than the first, when $(R' - L') - (R - L)$ is *positive.* In this case, therefore, *C. S.* is 1.8 div. longer than *T.*

With the microscope of Comparator No. 1, however, this order is reversed. In that case, the second standard is shorter than the first, when $(R' - L') - (R - L)$ is *positive.*

(*d.*) *Comparison by means of Stops, and with two Microscopes.*

(1.) The standards to be compared are placed side by side upon the table S, and the carriage is placed at a convenient point upon E.

(2.) The plates K and I are adjusted so that the defining lines of the first standard to be compared shall fall under the microscopes M and M^1.

(3.) The stops H and H^1, having been brought into contact with the plates K and I, are securely clamped.

(4.) After the adjustments for position and for focus have been made, successive contacts of the plates K and I are made with the stops H and H^1, and the microscopes M and M^1 are read for each contact. These operations having been repeated for each standard to be compared, the values of $(M^1 - M)$ when reduced to a common unit and compared *inter se*, will give the relations required.

It is the experience of the writer that the microscope carriage can be brought into actual contact with the stops, by means of the rack and pinion movement, with greater certainty than it is possible to make a coincidence of the micrometer thread with the defining lines of the standard. The following test has been frequently tried, and always with the most conclusive results. With a quarter-inch objective, in which the value of one division of the micrometer screw is only 0.11 μ, a series of one hundred successive contacts with the stops were made without disturbing the position of the micrometer thread. The number of cases in which the deviation from the mean of the first two or three readings of the micrometer was perceptible to the eye were noted, and the amount of the deviation was estimated in terms of the apparent width of the micrometer thread. One hundred readings of the micrometer thread were then taken for coincidence with the defining line of the standard, the plate K remaining stationary. A comparison of the results obtained in this way has always been found to be in favor of the stop contacts. With a little experience on the part of the observer, the stop method admits of the highest degree of precision. It is the experience of the writer, that one hundred successive contacts may be made, in which another observer at the microscope will be unable to detect the slightest deviation from constancy.

The stop method, also, has the great advantage over all other methods, that it allows perfect freedom in the adjustment of the microscope for focus, at any time during the comparisons. It is only required that the stops shall remain fixed during the two or more comparisons.

But in order to meet the objection which is sometimes urged against the certainty of actual contact, an electro-magnetic attachment has been added to the stops H and H^1, by which the plates K and I are

securely locked *after* the contact has been made by the rack and pinion movement. By means of a screw, shown at the left of H, the core of the magnet is adjusted at the proper distance with respect to the armature shown at the left of K, after contact has been made between the stops. A battery is employed of sufficient force to move the carriage K, when the magnet is one eighth inch distant from the armature.

(*e.*) *Comparison by means of two Microscopes placed in a Horizontal Position.*

(1.) By means of two angle-plates, microscopes M and M^1 are made to occupy a position at right angles to the position shown in Fig. 1.

(2.) The standards to be compared are placed upon the table S, supported at their neutral points, and with their graduated surfaces in a vertical plane.

(3.) The comparisons are then made in the manner described under (*a*).

By this arrangement, the apparatus becomes a vertical comparator. The intersection of the transverse line with the defining lines being in the plane of the neutral axis, the deflection will be inappreciable, even if the bar is not supported at its neutral points. The first suggestion of this form of apparatus is generally credited to Neuman, and the first construction to Wilde, who has given a full description of the vertical comparator, constructed under his supervision, in the Repertorium für Experimental-Physik, Vol. XIII. p. 567. But a comparator of this form, invented by Lane, had been in use for several years previous to this time in the office of the U. S. Coast Survey at Washington.

Under the fifth requirement the following methods are employed. The frame of the stop V, Fig. 4, is securely fastened to the bed-plate of the comparator. It has a vertical adjustment by means of gibbed ways. The stop U is firmly attached to the plate K. This stop has a vertical adjustment also; but the line V U was made parallel with the cylindrical ways by the makers.

For observations of this class, the plate N, Fig. 4, which carries the microscope M^1, occupies a position the reverse of that shown in the cut.

The line bar to be compared is first placed upon the table at a known distance outside of the line U V. The adjustments are then made as follows: —

(1.) The stops U V are set at the same height above the table.

(2.) The line bar is then adjusted so that the horizontal line is parallel with the cylindrical ways, and is at the same time in the focal plane of the objective. The means of adjustment in three planes are not shown in the figure, but they are the same as those already described. The frame which carries the lever movements is fitted to the bed-plate by the dowel pins O O.

(3.) The plate K is moved upon the cylindrical ways till a contact between U and V takes place. While the plate remains in this position, micrometer M^2 is read for coincidence with the defining line at this end of the bar.

(4.) The carriage having been moved to the other end, the end-measure standard is placed in position between the stops U and V.

(5.) After firm contacts have been made between the stops and the defining surfaces of the end-measure standard, through the hand-wheel R of the rack and pinion, microscope M^1 is read for coincidence with the defining line at that end of the bar. The difference in the two readings will be the difference in the length of the two standards, provided the cylinders have no horizontal curvature.

(6.) In order to eliminate the effect of horizontal curvature, the comparison is again made with the line bar, placed at the same distance inside of the line U V.

Good results have been obtained by this method, but it is of course open to two objections; first, that since the bar must be moved in order to secure contact at V, the pressure at the two ends is unequal; and second, that the force required to make the contacts secure, may produce indentations in the defining surfaces of the standard bar. It will be seen from the series of observations with the end-measure standard in melting ice, detailed on pp. 345–359, that the second objection will not hold if the steel is properly tempered. The difficulty, if it exists, is easily remedied by the use of protecting pieces having parallel surfaces. The first objection is a more serious one. It is certainly open to criticism in a theoretical view, but the results shown on page 350 seem to indicate that this objection is not a serious one in actual practice.

A second method of comparing line with end measures will need to be described without the aid of a drawing. The apparatus is attached to table S, Fig. 1.

Three requirements must be met in the successful comparison of line-measure with end-measure standards: —

(1.) The position of the end-measure standard must not be disturbed during the comparison.

(2.) If only one microscope is used, either the movement of the microscope along the horizontal defining line must be in the line of the stops, or there must be a convenient way of placing the line standard at an equal and constant distance on either side of the line of the stops, without the necessity of adjustment for parallelism and for focus. If two microscopes are used, means of adjustment in three planes become necessary.

(3.) The pressure of the stops at each end must be equal and constant.

These three conditions are met in the following arrangement. Upon a table attached to S, Fig. 1, two slides move freely upon cylindrical ways of rather small diameter, but they can be firmly secured to the cylinder at any point by clamps. A second slide is mounted upon each of these plates, which has a movement of about four inches upon a cylinder and a flat way. The parallelism of these upper plates with the plane described by the carriage K, is secured by an adjustable vertical piece which passes through the plate near one edge and rests upon the plane way, which has also a vertical adjustment. There are two of these plane ways, placed equidistant from the cylindrical way on either side. The carriage can therefore be reversed by simply lifting it from its semicircular bearings. These upper slides terminate with tempered steel stops, which were set by the maker in a line with each other, and in the vertical plane which passes through the axis of the cylinder. The forward motion is given by means of an adjustable spiral spring, while a lever and stop control the backward motion.

An adjustable graduated bar is firmly attached to each of these plates, which has its upper surface nearly in a line with the stops.

With this arrangement of stops it is possible to compare line with end measures by the aid of either one, two, or three microscopes. The method by two microscopes has decided advantages over any other tried by the writer. The explanation, therefore, will be limited to this method.

(1.) The two double-slide plates are placed in such a position upon the main cylindrical ways that the distance between the spring-stops is e. g. about one meter.

(2.) The end meter, supported at its neutral points; is placed between the stops, and an equal pressure upon each end is given by means of the adjustable springs.

(3.) Plate K is then placed nearly opposite one end of the meter, and plate I nearly opposite the other end.

The micrometer line of M^1 must now be placed exactly in the verti-

cal plane with the contacts of the stops at one end, and the micrometer line of M must be placed in the same vertical plane with the contacts of the stops at the other end. This is accomplished in the following manner. The stop-plate is thrown back by means of the controlling lever, and a rectangular block of metal is placed between the stop and the defining surface of the bar. This block has two of its faces exactly parallel and one inch apart. The upper surface is supposed to be in the same plane as the upper surface of the short graduated reference-bar attached to the stop-plates. The upper surface has a slight projection at each end, which allows defining lines one inch apart. This surface is bisected approximately by a horizontal and a perpendicular defining line, and a perpendicular line is drawn at each end exactly half an inch from the line at the middle point.

The first step is to set the micrometer M^1 for coincidence at the intersection of the cross lines upon the plate. The block is then turned 180° and a second reading of M^1 is taken. One half of the difference between the two readings will measure the distance of the defining line on the block from the middle point between two stops when contacts are made with the end surfaces. If the plate K is moved forward a distance equal to

$$\left[\tfrac{1}{2}\text{ inch} + \frac{M_1^1 + M_2^1}{2}\right],$$

the micrometer line of the microscope will be in the perpendicular plane which passes through the point of contact formed when the two stops are brought together by the removal of the block.

In the same manner the reference line of micrometer M is brought into coincidence with the plane passing through the point of contact of the stops at the other end. The distance between the micrometer lines of the two microscopes is now equal to the length of the end-measure standard.

(4.) The line-measure standard to be compared is now placed upon the table S, and the distance between the defining lines of the meter is compared with the distance indicated by a constant plus the readings of the microscope micrometers. The adjustments required are the same as those described under division (*a*), page 302.

The comparison of short-end measure gauges can be expeditiously made by means of the graduated reference-bar attached to the stop-plate. For this purpose the two lower slides are brought near enough together to allow the stops of the upper plates to be brought into contact. Both slides are then securely fastened to the cylindrical ways upon which they rest, and one of the stop-slides is securely clamped

also. After contact has been made between the free and the fixed stops, the micrometer line of the microscope is brought into coincidence with the defining line at one end of the bar attached to the free stop-plate. The end gauge is then placed between the stops, the free slide being moved back the required distance in order to allow the insertion. During this operation the microscope remains undisturbed, and the graduated bar passes under it without disturbance of the focal plane. The difference in the readings, therefore, for the two positions of the free stop-plate, i. e. before and after the insertion of the end gauge, measures its deviation in length from the line standard. If three microscopes are mounted upon plate K, and one of the stop-plates is attached to the front face of the carriage in such a manner that the two stops are parallel with the cylindrical ways, the method of comparison already described is applicable, and conditions (1) and (2) are both fulfilled.

The sixth condition is met in two ways.

(1.) By the use of the two stops, H, H^1, Fig. 4.

The stops are first set approximately at the distance apart indicated by the subdivisions to be compared. The method of comparing each space with the constant distance between the stops is the same in every respect as that described already. The following example is given in illustration.

COMPARISON OF INCH SUBDIVISIONS OF $G_2^{a_2}$.

Space.		L	R	$R-L$	$(R-L)$ *minus* Constant.	Σ
1	5 rev.	0.4 div.	3.8 div.	+3.4 div.	—2.7 div. = —1.4 μ	—1.4 μ
2	....	0.0 div.	8.3 div.	+8.3 div.	+2.2 div. = +1.1 μ	—0.3 μ
3	....	0.6 div.	5.7 div.	+5.1 div.	—1.0 div. = —0.5 μ	—0.8 μ
4	....	1.3 div.	7.9 div.	+6.6 div.	+0.5 div. = +0.3 μ	—0.5 μ
5	....	1.0 div.	9.7 div.	+8.7 div.	+2.6 div. = +1.3 μ	+0.8 μ
6	....	1.1 div.	5.5 div.	+4.4 div.	—1.7 div. = —0.8 μ	+0.0 μ
			Mean	+6.1 div.		

In this comparison the distance between the stops was 6.1 div. less than the average value for one inch. When this constant is subtracted from the separate values of $R - L$, the residuals represent the relative values of the separate subdivisions. With the microscope of Comparator No. 1 a positive residual indicates that the measured space is too short. In this case the first space is 1.4 μ relatively too long, and the second 1.1 μ relatively too short. In order to obtain the error of any line reckoned from the first line, we must sum the residuals algebraically. Here the first five inches are 0.8 μ too short.

(2.) By the use of two microscopes.

It will be seen in Fig. 1 that there are two carriages, S and S^1, which traverse independent ways. S^1 is supposed to be in front of S. Two arms are attached to the upper surface of table S^1 (not shown in the figure), which project about half-way over the surface of S, allowing a slight clearance between the two surfaces. The standard whose subdivisions are to be compared is placed upon the table S, and a bar having upon its surface a graduated space, X, approximately equal to one of the subdivisions of the first bar, is placed upon the projecting arms attached to S^1. Both microscopes are now secured to the carriage K. M^1 is adjusted for coincidence with the initial line of the standard, which is placed upon S, and M is adjusted for coincidence with the corresponding line upon the other bar. The carriage K is moved toward the left, and a second coincidence of M is made with the other defining line. A comparison of the two readings of M^1 for the two positions of M will give the relation between the two spaces compared. M^1 is then adjusted for coincidence with the second line, and the carriage S^1 is carried back by the rack and pinion D till the first line is again under M. The second subdivision can now be compared with the same constant space, X, as before. It is obvious that, by the proper disposition of the two bars with respect to M and M^1, subdivisions of any magnitude whatever can be compared. It is to be noted, however, that the results obtained in this way are subject to errors due to the horizontal curvature of the cylindrical ways.

(3.) For long distances, and for any distance exceeding one decimeter, more reliable results can be obtained by the use of two microscopes in a fixed position. The various adjustments required in order to bring the defining lines of any subdivision into coincidence with M and M^1 have already been described.

Description of Comparing-Rooms.

The comparator first described is mounted upon isolated brick piers in the cellar of the Observatory. Through the liberality of the Director of the Observatory a small room in the shape of a trapezium was partitioned off for this purpose. This comparator was at first mounted upon stone piers mounted upon the clock pier in the prime-vertical room of the Observatory. But notwithstanding the great size and stability of this pier, it was found impossible to make comparisons except very early in the morning before the disturbance from passing teams began. This pier rests upon a layer of blue clay having a large

inclination to the horizon. Beneath there is a layer of sand. The tremors communicated to the clock pier through this combination were found to be surprisingly great. The tremors produced by ice-carts at the distance of 1,000 feet rendered it impossible to make exact comparisons. The heavy steps of an assistant could be counted to the distance of 100 feet, by noticing the effect of the concussion upon the pier, which was in turn communicated to the surface of the bar examined under the microscope.

Heat is communicated to this comparing-room from the furnace in the cellar by a pipe which enters the room near the ceiling and nearly over the comparator. Under certain circumstances the temperature in this room may be kept under very good control, but great care has been found necessary in this regard. The comparator is directly in the line of the windows. By opening both windows during the night, the temperature within the room is found to be reduced nearly to the temperature of the outside air. By making the observations with this comparator on cloudy days and early in the morning, much better results have been obtained than were expected.

By the liberality of the President of Harvard College, comparing-room No. 2 was fitted for the reception of the Universal Comparator. It is situated in the basement of Harvard Hall. The dimensions of the room are 12 feet in length, 9 feet in width, and 8 feet in height. The brick walls which surround the room are twelve inches in thickness. The brick piers upon which the comparator rests extend to a depth of eight feet. The walls, the ceiling, and the floor are all double-planked, with two intervening air spaces. Between the inner and the outer partition there is a layer of rosin-soaked paper. The room has double windows and double doors. The steadiness of the temperature within the room may be inferred from the record on pp. 366–371. During the summer of 1882 the extreme change of the Fahrenheit thermometer for eight weeks was 1°.3. During the same time the daily variation in the temperature of the outside air often amounted to 25°.

Between April 14 and June 1, 1883, the extreme range was 0°.75 C. On June 3 the temperature of the room was raised by artificial means. By June 5 the temperature had reached the stationary point, viz. 17°.07 C., and between this date and June 28 the rise was only 0°.96. On July 6, the reading of *Y* 61 was only 18°.48, notwithstanding the average temperature of 31° for the two preceding days. On July 7, the room was exposed to the open air.

An attempt has been made to produce desired variations of temper-

ature by the following arrangement, which has been found to be moderately successful. Sheets of galvanized iron three feet in width, after having been fastened at the edges by soldering, and riveted together at intervals of twelve inches, were fastened to the inner walls of the room in such a manner that there is a gradual decline in their height above the floor. They are connected together by rubber tubing. In the opposite room a tank is arranged for either hot or cold water, which is connected with the reservoir at the highest point. At the lowest point there is an outlet through the walls of the room. Either hot or cold water entering this reservoir of narrow section slowly percolates through the enclosed space, and flows off through the outlet, maintaining a nearly constant supply of either hot or cold air within the room. The reservoir holds about five gallons, and the amount of surface exposed on one side is about 100 square feet.

This comparing-room has one fault in construction which has given considerable trouble, and which it has been found impossible to remedy entirely. In order to be able to raise the comparator as high as the window, it was found necessary to give the piers a height of four feet. The head of the observer is therefore very near the ceiling. This difficulty has been partly remedied by a large trap-door in the ceiling, which allows the heat developed by the presence of the observer to enter the space between the partitions.

Description of the Thermometers employed in the Comparisons.

Throughout the entire series of observations the Yale College standard has been adopted. In the earlier part of the work a Fahrenheit thermometer graduated to fifths of degrees was used. It is designated *O*. It was purchased of a dealer in New York at a low price, but it was found to be an exceptionally fine instrument. Its error was determined by Dr. Waldo, by comparison with the Yale standard. It was afterwards carefully compared with Cassella No. 3235, which had been rigorously compared with the air thermometer of Professor Rowland by Mr. S. W. Holman of the Massachusetts Institute of Technology. The independent comparisons of Dr. Waldo and of Mr. Holman agree in giving the same tenth of a degree for every point compared. The corrections adopted are given below.

T	O	3235	T	O	3235
32°	$-0^\circ.74$	$-0^\circ.80$	60°	$-0^\circ.90$	$-0^\circ.94$
35	—0.70	—0.81	65	—0.98	—0.95
40	—0.70	—0.84	70	—0.98	—0.97
45	—0.71	—0.86	75	—0.92	—0.97
50	—0.75	—0.89	80	—0.93	—0.98
55	—0.79	—0.91			

As a check upon the observations with *O*, a Centigrade thermometer, marked No. 1, was read in connection with it. The corrections to No. 1 were found from a comparison with *O* to be as follows: —

T	No. 1	T	No. 1	T	No. 1
0°	$-0^\circ.50$	12°	$-0^\circ.94$	21°	$-0^\circ.72$
3	—0.67	15	—0.97	24	—0.50
6	—0.78	18	—0.90	27	—0.63
9	—0.90				

The standard designated *Y* 61 was received from Dr. Waldo in October, 1882. Since this date all comparisons have been made with reference to this standard. The following Report accompanied the standard: —

"THE OBSERVATORY OF YALE COLLEGE. — THERMOMETRIC BUREAU.

"*Examination of the Y. O. S. Thermometer, No.* 61, *made by Tonnelot.*

"1st. This thermometer has been examined in a vertical position with the metallic scale and tube immersed in water having the temperature of the bulb.

"2d. When the correction is + it must be added to the thermometer reading, and when — it must be subtracted. For example, suppose the thermometer to register 81°.0 and the respective tabular corrections at 72° and 92° to be —0°.5 and —0°.7, then the corrected reading of the thermometer would be 81°.0 — 0°.6 = 80°.4.

"The theoretical mercurial standard thermometer to which this instrument has been referred, is graduated by equal volumes upon a glass stem of the same dimensions and chemical constitution as the Kew standards 578 and 584. The permanent freezing point is determined by an exposure of not less than forty-eight hours to melting ice, sup-

posing the temperature of the standard has not been greater than 25° C. = 77° F. during the preceding six months. The boiling point is determined from the temperature of the steam of pure water at a barometric pressure of 760 mm. = 29.922 in. (reduced to 0° C.) at the level of the sea and in the latitude of 45°. This standard coincides with the perfect gas thermometer within 0°.1 F. for temperatures between zero and 212° F.

"LEONARD WALDO,
Astronomer in Charge.

"New Haven, Conn., March–October, 1882.

"YALE OBSERVATORY STANDARD NO. 61. — O. T. S., OBSERVER.

Reading of Y. O. S. 61.	Correction to be applied.	Depression at 0° for 100° Elevation.	Reading of the 0° point when horizontal.	Correction depending on Calibration.
°	°	°	°	°
0.0 C.	0.00	—0.20	+0.04	0.00
5.0	—0.06			
10.0	0.00			
12.5				+0.17
15.0	+0.02			
20.0	+0.07			
25.0	+0.12			+0.33
30.0	+0.16			
35.0	+0.18			
37.5				+0.36
40.0	+0.13			
45.0	+0.20			
50.0	+0.12			+0.41
55.0	+0.14			
60.0	+0.20			
62.5				+0.54
65.0	+0.16			
70.0	+0.28			
75.0	+0.21			+0.45
80.0	+0.20			
85.0	+0.13			
87.5				+0.29
90.0	+0.02			
95.0	—0.02			
100.0	+0.07			0.00

"1. The first column gives the scale readings on the thermometer.

"2. The second column gives the sum of all the corrections to be applied at the points of the scale indicated in 1, to reduce the readings to the standard of this Observatory.

"3. The third column gives the depression of the zero point caused by heating the standard to 100° C.

"4. The fourth column gives the reading of the zero point when the standard is inclined 90°.

"5. The fifth column gives the correction depending on the interior figure of the standard column."

It will be noticed that the report of Dr. Waldo gives only the corrections of *Y* 61 between the limits 0° and 100°. In the reduction of the observations at temperatures below 0° for the determination of the absolute coefficients of expansion of the bars under consideration, I found a constant tendency to a positive correction, on the supposition that the coefficient was constant. For example, instead of finding the constant difference between bars *T* and *S* in melting ice to be 75.1 div., as determined from the equations of condition on the following pages, I found continually diminishing values, until for $Y = -11°$ the value was 52.6 div. In the case of bar *C. S.* a similar result was found. The coefficients were now assumed to be a constant, and the corrections to the thermometer required to make all the values of $S - T$ and $S - C.S.$ below 0° agree with the values derived from observations above 0° were computed, with the following results.

Corrections to Y 61.

Y 61	From *T*	*Y* 61	From *T*
°	°	°	°
—10.8	+0.35	—4.5	—0.02
—10.4	+0.18	—3.6	—0.01
— 9.9	+0.20	—2.1	+0.06
— 7.9	+0.13	—1.2	—0.04
— 7.0	+0.03	—0.7	—0.07
— 6.1	+0.10	—0.5	+0.05
— 4.6	+0.03	—0.4	+0.01

A similar series of corrections was derived from bar *C. S.* Smooth curves were now drawn through the points determined by these residuals, and the following corrections were obtained. Afterwards Mr. S. W. Holman, of the Institute of Technology, compared *Y* 61 with an alcohol standard thermometer by Baudin. His results are given below. It should be noted that Mr. Holman's values were derived before he had any knowledge of the results obtained by the writer.

Y 61	From *T*	From *C. S.*	Mean.	Holman.	Adopted Corrections.
°	°	°	°	°	°
— 1	—.02	+.09	+.03	+.03	+.03
— 2	—.02	+.14	+.06	+.06	+.06
— 3	—.01	+.18	+.09	+.09	+.09
— 4	+.01	+.22	+.12	+.13	+.13
— 5	+.04	+.22	+.13	+.13	+.13
— 6	+.06	+.22	+.14	+.14	+.14
— 7	+.14	+.22	+.18	+.15	+.16
— 8	+.20	+.23	+.22	+.18	+.20
— 9	+.30	+.24	+.27	+.24	+.25
—10	+.40	+.26	+.33	+.31	+.32
—11	+.48	+.32	+.37	+.39	+.38

The writer is using at the present time in connection with *Y* 61 a spirit thermometer made by Mr. J. S. Huddleston of Boston. The length of each degree Centigrade is about 2.5 centimeters, and the length of the column of (colored) alcohol is about thirty-nine inches. The complete determination of the errors of this thermometer will need to be deferred till the coming winter; but according to the present indications it is an instrument of extraordinary precision. The greatest deviation thus far observed from the corrected readings of *Y* 61 is about 0°.06.

Description of Microscopes.

Values of One Revolution of Micrometer Screws.

The measuring microscope used in connection with Comparator No. 1 has a tube fourteen inches in length. The micrometer was made by Powell and Leland.

Nearly all of the observations have been made with a Tolles four-system inch. The illumination is invariably obtained by the use of a prism inserted between the two front lenses, a device known as Tolles's opaque illuminator. The principle of this illuminator is so often stated incorrectly, that it is well to restate it here. The focus of the rays of light which pass through the prism is a little outside of the focus of the objective itself. The image under the objective, therefore, is *within* a cone of diffused light, the axis of the cone being in the line of collimation of the objective. This condition, however, requires a somewhat careful adjustment of the prism when it is set by the maker.

The illumination of polished metal surfaces is simply perfect. Sky illumination gives better results than artificial illumination. The distance of the objective from the opening through which the light passes

may be as great as 100 feet. It is only necessary that the plane face of the prism shall be directed towards the source of light. In order to get the best results, the power of the objective should be as great as the equivalent of an inch lens. But if an objective of great working distance is required, a large prism may be mounted in front of the lens. The writer has made use of this method of illumination for the microscopes of the Harvard College Meridian Circle with good success. The focal length of these objectives is four inches.

The following are the results of the various determinations of the values of the micrometer screws.

Comparator No. 1, Microscope A, 1 inch objective.		
1880, Apr. 25	1 div. =	0.505μ
1880, Apr. 26	"	0.504
1882, June 8	"	0.503
1882, Oct. 15	"	0.504
1883, Feb. 26	"	0.505
1883, Feb. 28	"	0.504
Mean,		0.504

Universal Comparator, Microscope A, 1 inch objective.		
1882, Oct. 15	1 div. =	0.438μ
1883, Mar. 23	"	0.441
1883, Mar. 29	"	0.441
1883, Apr. 5	"	0.442
1883, Apr. 6	"	0.440
Mean,		0.440

It will not escape attention that no mention has thus far been made of any arrangement for protecting the standard bars to be compared from the effect of the increase of temperature due to the presence of the observer in the comparing-room. The omission has not been accidental.

The common impression, that the effect of such increase of temperature will be immediately apparent, is erroneous. If the meter is traced upon a thin ribbon of steel, any increase of temperature amounting to 10° will produce the change in length which its coefficient of expansion demands, within 15 or 20 seconds; but if its mass be increased two hundred fold, a change in temperature of the same amount will require from one to two hours for its normal action.

The preparatory work which needs to be done with every bar which is to receive standard graduations is the determination of the time required for a given change of temperature to produce its full effect. This time is a function of the shape and the mass of the bar. It is proposed during the approaching winter to make an exhaustive study of this element for the bars under consideration. The few observations which have already been made seem to show that, if these bars are quickly removed from a temperature of 32° to a constant temper-

ature of 62°, the times required for them to reach their normal length will be about as follows: —

T	requires from 15 to 20 minutes.
$C. S.$	requires from 30 to 40 minutes.
R_1	requires about 2 hours.
R_2 and G	require about 4 hours.

Conversely, the times within which an increase of temperature amounting to one degree, *as indicated by the thermometer*, will be inappreciable in the comparisons, are about as follows: —

T	4^m
$C. S.$	10^m
R_1	12^m
R_2	18^m
G	22^m

The Tresca bar, T, seems to possess decided advantages over every other form for the usual conditions under which observations are made. However sudden the change of temperature, and however great the amount of the change, the thermometer will indicate the true length of the bar, if an observed temperature can remain constant for about eleven minutes after the mercury reaches the stationary point. On the other hand, standards having a large cross-section are to be preferred, if a constant temperature can be maintained for several hours, since the effect of any change can be neglected for a considerable time.

Instead, therefore, of arranging protecting screens for the comparator, I have endeavored to arrange the adjustments in such a manner that a complete set of comparisons can be made before the heat developed by the presence of the observer could produce any effect. In the comparisons described in this paper, the bar T, having the least mass, is placed in front, and the microscopes are left in adjustment upon the defining lines from the previous observation. The time required to complete the observations upon this bar is not, therefore, over one or two minutes. Bar $C. S.$, being the next in order of mass, is observed next in order of time, and bar R_2, having the greatest mass, is observed last. *But after the observations are completed, the comparing-room remains closed for at least four hours.* During this time, the increase of temperature due to the presence of the observer, which is

usually about 0°.4 C., will have become absorbed in the general temperature of the room, and the several bars will have reached a state of rest as far as this special increase is concerned.

We are now prepared to enter upon an examination of the standard prototypes described in this paper. The various comparisons will be given nearly in the order in which they were made.

Comparison of Yard $R_2^{a_2}$ with "Bronze 11" at Washington.

Professor Hilgard having kindly consented to undertake the comparison of the bronze yard $R_2^{a_2}$ with "Bronze 11," Assistant Edwin Smith was assigned to this work. The observations by Mr. Smith were made with the Lane vertical comparator. At his request, the writer made independent micrometer readings, after all the adjustments had been made, but there was a complete avoidance of any knowledge of the results obtained.

The full text of Professor Hilgard's report will be found in the report by the writer to the Pratt and Whitney Company, relative to the standards constructed for them; but the portion relating to R_2 is given here.

Date.			Temperature. F.	"Bronze 11" — R_2.	
				Smith.	Rogers.
1881	h	m	°	in.	in.
Jan. 26	9	49 A. M.	56.20	—.000116	—.000100
"	11	21 "	56.15	—.000114	—.000089
"	1	28 P.M.	56 20	+.000004	+.000022
"	3	11 "	56.35	—.000060	—.000084
Jan. 27	9	39 A.M.	55.90	+.000063	+.000052
"	11	24 "	55.55	—.000002	—.000014
"	1	23 P.M.	55.10	—.000140	—.000125
"	3	10 "	56.40	—.000133	—.000098
Jan. 28	9	36 A.M.	52.00	—.000064	—.000072
"	11	22 "	51.75	—.000041	—.000041
"	12	36 P.M.	51.30	—.000018	......
"	2	16 "	51.30	+.000015	......
Smith			54.37	—.000050	......
Rogers			54.98	......	—.000055

At 54°.70 F. "Bronze 11" — R_2 = —0.000052 inch.

Y — "Bronze 11" = +0.000088 "

$Y - R_2$ = +0.000036 "

I am indebted to the courtesy of Professor Hilgard for the opportunity of comparing R_2 with "Bronze 11" upon Comparator No. 1, which was sent to Washington for this purpose. Since subsequent comparisons were to be made with this comparator, it seemed important that the relations between these standards should be determined under the same conditions as those under which future investigations would be made.

For this work I was assigned to a room in the basement of the building, in which a pretty steady high temperature could be maintained. Afterwards the comparator was removed to a small observatory building in the rear, which was admirably adapted for the purpose. Here a nearly constant low temperature was maintained for three days. The comparator was then mounted again in its former location, and further observations were made at a nearly constant temperature, which had now become reduced to about 62°.

The following are the results of the comparisons.

Date. 1881	Observer.	Thermometer. °	"Bronze 11" *minus* $R_2^{a_2}$. in.
Feb. 1 A.M.	R.	35.2	—.000030
" 1 "	R.	35.2	+.000028
" 1 P.M.	R.	35.6	—.000002
" 1 "	R.	35.6	—.000083
" 2 A.M.	R.	30.9	—.000014
" 2 "	S.	30.9	—.000030
" 4 P.M.	R.	60.9	—.000015
" 7 "	R.	62.2	—.000059
		Mean....	—.000027

We have, therefore,

$$\text{"Bronze 11"} - R_2^{a_2} = -0.000027 \text{ inch.}$$

$$\text{"Bronze 11"} - Y = -0.000088 \text{ "}$$

$$R_2^{a_2} - Y = -0.000061 \text{ "}$$

Finally,

$$R_2^{a_2} + \frac{0.000036 \text{ in.} + 0.000061 \text{ in.}}{2} = Y.$$

Or, $$R_2^{a_2} + 0.000048 \text{ in.} = Y.$$

Since the metal in each bar has the same composition, it is assumed that they have the same coefficient of expansion.

Comparisons were also made between the Tresca meter and the Coast Survey meter "No. 49," which bears a known relation to the Berlin meter, and thence with the Metre des Archives.

The following equations of condition were obtained.

Date. 1881	"49" — T^{a_2}			$(\tau - 32°)$	"49" — T^{a_2} at 32°	Δa
Jan. 24	$+139.4\,\mu$	=	a	$-40.2\,b$	$+90.0\,\mu$	$-1.7\,\mu$
" 24	$+140.1\,\mu$	=	a	$-40.3\,b$	$+90.5\,\mu$	$-1.2\,\mu$
" 24	$+138.6\,\mu$	=	a	$-40.3\,b$	$+89.0\,\mu$	$-2.7\,\mu$
" 24	$+140.3\,\mu$	=	a	$-40.5\,b$	$+90.5\,\mu$	$-1.2\,\mu$
" 25	$+139.6\,\mu$	=	a	$-37.8\,b$	$+93.1\,\mu$	$+1.4\,\mu$
" 26	$+140.9\,\mu$	=	a	$-38.3\,b$	$+93.8\,\mu$	$+2.1\,\mu$
" 30	$+\ 96.2\,\mu$	=	a	$-\ 4.2\,b$	$+91.0\,\mu$	$-0.7\,\mu$
" 31	$+\ 96.0\,\mu$	=	a	$-\ 3.3\,b$	$+91.9\,\mu$	$+0.2\,\mu$
Feb. 1	$+\ 95.4\,\mu$	=	a	$-\ 4.1\,b$	$+90.4\,\mu$	$-1.3\,\mu$
" 2	$+129.1\,\mu$	=	a	$-30.4\,b$	$+91.8\,\mu$	$+0.1\,\mu$
" 6	$+131.0\,\mu$	=	a	$-29.4\,b$	$+94.8\,\mu$	$+3.1\,\mu$

Normal Equations.

$$+1386.6 = +11\,a - 308.8\,b \qquad b = +1.23\,\mu$$
$$-42076.8 = -308.8\,a + 11234.1\,b \qquad a = +91.7\,\mu$$

It will hereafter be shown that at 32°

$$T^{a_2} + 102.8\,\mu = A.$$

But
$$\text{“49”} - T^{a_2} = 91.7\,\mu.$$

Hence
$$\text{“49”} + 11.1\,\mu = A.$$

Förster gives for "49" the following relations: —

From the direct comparison of the Berlin meter with the Metre des Archives,

$$\text{“49”} + 21.4\,\mu = A.$$

From a comparison of the Berlin meter with the meter of the Conservatory,

$$\text{“49”} + 5.2\,\mu = A.$$

It will be seen that the value found in terms of the Tresca meter falls between these values.

Since the absolute coefficient of expansion of "49" has been determined both by Förster and by Pierce, the relation here obtained between T and "49" will yield a value for T. We have for "49," for each degree Centigrade,

Förster,	$18.69\,\mu$.
Pierce, direct,	$18.83\,\mu$.
Pierce, indirect,	$18.81\,\mu$.

Reducing the value $b = 1.23\,\mu$ to its equivalent for one degree Centigrade, we have the following values for the absolute coefficient of T: —

From comparison with Förster, $16.48\,\mu$.
" " Pierce, $16.60\,\mu$.
" " Pierce, $16.62\,\mu$.

It will be seen that these values are somewhat larger than the value derived from the observations which follow; but it can hardly be expected that the correct relation between T and "49" could be obtained from the limited number of observations here given.

COMPARISON OF THE FROMENT END-METER F_e WITH T^{a_2} WITH COMPARATOR NO. 1.

Date. 1880.	T (Fahr.)	$T^{a_2} - Fe$	* $b = 3.40\,\mu$ $T^{a_2} - Fe$ at 32° Fahr.	Mean values.
Nov. 19	47.6°	+ 53.3 μ	+106.3 μ	
" 22	47.1	+ 54.6 μ	+105.9 μ	
" 24	19.1	+158.0 μ	+114.1 μ	
" 25	33.9	+109.2 μ	+115.7 μ	
" 28	76.1	— 44.8 μ	+105.1 μ	
" 29	45.3	+ 66.0 μ	+111.6 μ	109.8 μ
Dec. 1	57.3	+ 19.1 μ	+105.1 μ	
" 17	37.4	+ 90.0 μ	+108.4 μ	
" 19	35.4	+ 10.2 μ	+113.8 μ	
" 21	38.9	+ 84.4 μ	+107.9 μ	
" 22	46.1	+ 58.8 μ	+106.7 μ	
" 24	34.6	+105.6 μ	+114.4 μ	109.4 μ
" 26	49.6	+ 51.9 μ	+111.7 μ	
" 27	51.5	+ 40.8 μ	+107.1 μ	
" 28	39.8	+ 86.9 μ	+113.4 μ	
" 29	19.2	+157.4 μ	+113.9 μ	
" 31	5.0	+204.4 μ	+112.6 μ	
1881. Jan. 4	52.9	+ 34.9 μ	+106.0 μ	110.8 μ

We have, therefore, for 0° C.,

$$T^{a_2} - F_e = +110.0\,\mu.$$

But $F_e - A = +\ 8.4\,\mu.$

Whence $T^{a_2} - A = +101.6\,\mu.$

According to Tresca $T^{a_2} - A = +102.8\,\mu.$

Diff. $= 1.2\,\mu.$

* This value of b is the final value derived from the investigation on the following pages.

COMPARISON OF THE TRESCA METER $T_1^{a_2}$ WITH THE LINE-METER F_e WITH COMPARATOR NO. 1.

1881.	$(\tau - 0^\circ)$ C.	$T^{a_2} - Fe$	$b = 3.40\,\mu$ at 0° C. $T^{a_2} - Fe$
Apr. 25	+16.5	+ 3.2 μ	+104.2 μ
" 26	+12.3	+25.6 μ	+101.1 μ
" 27	+13.1	+23.2 μ	+103.4 μ
" 27	+15.7	+ 6.8 μ	+102.9 μ
" 28	+11.5	+32.4 μ	+102.8 μ
" 28	+13.6	+18.3 μ	+101.5 μ
" 29	+10.8	+36.0 μ	+102.0 μ
" 29	+18.7	—12.0 μ	+102.4 μ
May 2	+10.2	+39.4 μ	+101.4 μ
" 2	+12.5	+34.4 μ	+100.9 μ

We have, therefore, for 0° C.,

$$T^{a_2} - F_e = 102.3\,\mu.$$

But $$T^{a_2} - A = 102.8\,\mu.$$

Hence $$F_e - 0.5\,\mu = A.$$

This relation has an importance far beyond any ordinary comparison of standards, since the centimeter derived from the Froment line-meter is the unit upon which Angstrom's wave lengths depend. It is besides the basis of nearly all the later physical investigations undertaken in France. If, therefore, the transfer of this meter to the surface of bar F is assumed to be without error, the correspondence of the whole unit with the Metre des Archives is nearly perfect.

COMPARISON BETWEEN METERS T AND R_2.

After my return from Washington, two series of comparisons between these standards were instituted, one series with Comparator No. 1, and the other with the Universal Comparator. In the first series two microscopes were used, being attached to the carriage. The bars were placed at a distance $2x$ centimeters apart, and observations were made for the positions $+x$ and $-x$.

For $x = +3.0$ cm.

Date. 1881.	$(\tau - 62.^\circ0)$ F.	$T^{a_2} - R_2^{a_2}$	At 62°.0 F. $T^{a_2} - R_2^{a_2}$
Feb. 14	+24.1	+177.7 μ	+164.0 μ
" 15	+23.5	+177.2 μ	+163.8 μ
" 16	—12.2	+152.2 μ	+159.2 μ
" 17	—13.2	+151.0 μ	+158.5 μ
" 20	+32.5	+185.4 μ	+166.9 μ
" 21	+22.2	+172.2 μ	+159.6 μ

Date. 1881.	$(\tau - 62.^\circ 0)$ F.	$T^{a_2} - R_2^{a_2}$	$T^{a_2} - R_2^{a_2}$
Feb. 22	—13.7	+145.5 μ	+153.3 μ
" 23	—13.7	+142.8 μ	+150.6 μ
Mar. 16	— 8.9	+147.5 μ	+152.5 μ
" 23	—13.6	+144.6 μ	+152.3 μ
" 24	— 9.7	+154.0 μ	+165.2 μ
" 25	+26.8	+178.9 μ	+163.7 μ
" 29	—11.8	+155.0 μ	+161.7 μ
" 30	—13.2	+151.3 μ	+158.8 μ
" 31	—15.4	+151.0 μ	+159.7 μ
Apr. 3	+29.3	+187.2 μ	+160.5 μ
" 8	+ 6.9	+165.0 μ	+161.1 μ
" 10	+ 8.9	+167.6 μ	+162.6 μ

For $x = -3.0$ cm.

Feb. 24	—16.3	+165.5 μ	+174.7 μ
" 25	+35.9	+197.8 μ	+177.4 μ
Mar. 11	+ 4.8	+174.7 μ	+172.0 μ
" 13	+23.1	+183.6 μ	+170.5 μ
" 14	+19.6	+186.1 μ	+175.3 μ
" 15	+22.4	+189.3 μ	+176.6 μ
" 16	— 9.0	+163.2 μ	+163.7 μ

We have, therefore,

$$\begin{aligned} & \qquad\qquad\quad T^{a_2} - R^{a_2} \\ \text{For } x = +3.0 \text{ cm.} &= +159.7\ \mu. \\ x = -3.0 \text{ cm.} &= +172.9\ \mu. \\ \text{Mean} &= +166.3\ \mu. \end{aligned}$$

With the Universal Comparator.

EQUATIONS OF CONDITIONS BETWEEN T^{a_2} AND $R_2^{a_2}$.

(With 1-inch objective.)

1 div. $= 0.440\ \mu$

Date. 1882.	$T^{a_2} - R_2^{a_2}$			$(\tau - 62^\circ)$ F.	At 62° F. $T^{a_2} - R_2^{a_2}$	No. Obs.
Apr. 26	+401.5 div.	=	a	—16.30 b	+380.1 div.	3
" 27	+396.8 div.	=	a	—13.41 b	+379.2 div.	4
" 28	+391.7 div.	=	a	—12.15 b	+375.8 div.	5
" 30	+392.3 div.	=	a	—10.47 b	+378.6 div.	5
May 3	+393.9 div.	=	a	— 9.42 b	+381.6 div.	5
" 12	+380.0 div.	=	a	— 1.24 b	+378.4 div.	3
" 20	+372.8 div.	=	a	+ 2.98 b	+376.7 div.	3
" 23	+375.2 div.	=	a	+ 4.64 b	+381.3 div.	3

Normal Equations.

$$+3104.2 = +8a - 55.37b \qquad b = -1.31 = -0.58\ \mu$$
$$-22062.0 = -55.37a + 823.30b \qquad a = +378.9 = +166.0\ \mu$$

It will be seen that the value of b derived from these equations is nearly identical with the final value adopted from the subsequent observations, that value being $-0.57\,\mu$. Combining the value of $T^{a_2} - R_2^{a_2}$ with the value derived from the observations with Comparator No. 1, we have

$$T^{a_2} - R_2^{a_2} = \frac{166.3\,\mu + 166.0\,\mu}{2} = 166.1\,\mu.$$

The value derived from the observations which follow is $165.5\,\mu$.

Determination of Absolute Coefficients of Expansion.

The earlier experiments made by the writer in the determination of the absolute coefficients of expansion of the bars under consideration were very unsatisfactory. At first, the attempt was made to determine the coefficient of one end-measure bar by subjecting it to wide ranges of temperature when submerged in water, and thence to determine the relative coefficients between it and the remaining bars whose coefficients were desired. This attempt was only moderately successful. While it was possible to obtain nearly a constant value for the coefficient of this bar by different combinations of the results on the same day, the deviation of the values for different days was considerably greater than should have been found. The chief difficulty consisted in the impossibility of keeping the entire mass of the water at the same temperature. No amount of agitation seemed to accomplish this.

A second method gave rather better results. It was found that an end-measure bar placed in the clock-room of the Observatory remained at a nearly constant temperatue, the change during twenty-four hours rarely exceeding a few tenths of a degree. It was also found to be possible to place this bar upon Comparator No. 1, and to make a comparison with any line-measure bar in a little less than one minute. During this time the change in the length of the bar was found not to exceed two or three divisions of the micrometer, even when the difference in temperature was as great as 20° Fahr. The plan of observation was as follows. The line bar was adjusted for position upon the comparator, and the reading of the microscope for the contact of the stops and the coincidence with the defining line at one end was made under favorable conditions for a steady temperature. The reading of the thermometer was also recorded. The end-measure bar was then brought into the room, placed in position between the stops, and the microscope carriage was set for contact between the stops and the end of the bar. From this point the remaining part of the observation could be made leisurely for any temperature lower than that of the

clock-room, since the relation of the microscope to the line bar would not be affected by a contraction in the length of the bar. Several pointings for coincidence with the other defining line of the meter were then made, after which the end-measure bar was returned to its former position. An interval of several hours was always allowed to elapse before a second comparison was made.

In the reduction of the observations, allowance was made both for the small deviation from a constant temperature in the end-measure bar, and also for the inferred rate of change between the time the bar was brought into the comparing-room and the time the observation for contact was made. Usually two or three contacts were made at intervals of thirty seconds, in order to obtain the observed rate of change.

Many observations of this kind were made, during the year 1881, for temperatures in the comparing-room ranging between 30° and 70°. The following values of the absolute coefficients were obtained. On account of the obvious imperfection of the method, it does not seem worth while to print the details of the work. It will be seen, however, that these values do not differ widely from those found from the method which will be presently described.

Absolute Coefficients.

For T, 16.58 μ. For R_1, 10.03 μ. For R_2, 17.55 μ.

The experience acquired in the observations thus far made was wholly in favor of the plan of deriving the coefficient of expansion of a line-bar for air contact, by comparison with an end-bar at a constant temperature. In no other way could an accurate comparison be so quickly made. But it is essential to the success of this method that the end-measure bar shall remain at a constant temperature during the entire series of comparisons with the line-standard at wide ranges of temperature. Only a few observations were needed to show that the end bar could be kept at an invariable temperature by keeping it constantly submerged in melting ice. It was found necessary to take only two precautions: first, that the bar should have a covering of ice about two inches in depth; and second, that the ice-trough should not remain in a high temperature for a longer time than about ten minutes. Experiment showed that, if there was too little ice in the trough in which the bar was submerged, the expansion beyond the normal length of 32° might amount to as much as 8 μ. It was also found that, if the ice-trough remained in the comparing-room at a high temperature for a time exceeding ten or fifteen minutes, an expansion amounting to about 5 μ was liable to occur, notwithstanding a vigorous stirring of the ice.

For the third series of observations the following preparations were made. A steel bar was prepared 6 mm. shorter than one meter. Steel plugs were inserted at each end, each having a projection of 3mm. The ends of the bar were then made as hard as possible, and the plugs were ground off till the defining surfaces were one meter apart. Careful and repeated observations showed that these end surfaces were truly parallel. The bar was then mounted in a trough of galvanized iron having a depth of 5 inches and a width of $5\frac{1}{2}$ inches. The trough was originally prepared for a yard, and, in the first series of observations which follow, the bar, which is placed near the bottom of the trough, extends 4.5 cm. beyond each end. At the conclusion of the first series the trough was lengthened, and in the second series the projection at each end was only 3 mm.

In planning the execution of this series of observations, there were certain difficulties about the use of the Universal Comparator for this purpose which could not be easily remedied; and it was found necessary to make use of Comparator No. 1. Since it had been found that the Tresca bar T was extremely sensitive to small changes of temperature, and since it was possible to maintain a nearly constant temperature in the comparing-room of the Observatory for short intervals of time, it was arranged to determine the absolute coefficient of T by comparison with the bar S in melting ice, and then to determine the relative coefficients between T and the remaining standards from comparisons with the Universal Comparator.

Partly as an experiment, however, simultaneous observations for the absolute coefficients of bars R_2 and $C. S.$ were made. A study of the tables which follow will show that the probable value of the results obtained is nearly proportional to the mass of the bars.

The various steps of the observations are as follows.

The line bar having been placed in position and adjustment upon the comparator during the afternoon, the comparing-room was closed till the next morning. During this time the temperature in the room generally became very steady. For low temperature observations, both windows were left open during the night. About sunrise the next morning the bar S, after having been submerged in melting ice for at least twenty minutes, was removed from the room in the cellar in which the pier of the Russian transit instrument is situated, and placed between the stops of the comparator. The comparison with the line-meters was then made in the manner already described. The ice-trough was then immediately removed to the cellar, and allowed to remain for a few minutes, when the operation just described was repeated, provided always that a steady temperature was maintained

meanwhile in the comparing-room. The high temperatures in the comparing-room were produced partly by heat from the furnace, but a steady supply of heat was furnished from a gas stove, which was placed directly beneath the bed-plate of the comparator.

After the first series was completed, the trough was lengthened, and some changes were made in the disposition of the bar *S*, by which its apparent length was slightly changed. During the second series, the position of the stops was not disturbed, and the adopted order of observations was strictly followed in every case. This series, therefore, may be supposed to have greater weight than the first one. An attempt was made to make an equal number of observations for slightly increasing and slightly decreasing temperatures, in order to eliminate to a certain extent the difference between the temperature of the thermometer and the temperature of the bar; for it must always be understood that the thermometer simply registers its own temperature and not necessarily the temperature of the bar upon which it is placed. This arrangement was, however, only carried out to a certain extent.

There is one point in connection with these observations to which particular attention is called. If the contact surfaces of the bar *S* become worn by continual use, it is evident that a constant error will be introduced, which will need to be investigated before the observations for the coefficients of expansion can be reduced. But it will be seen from the observations which follow, that there is no conclusive evidence of such a change during the entire series. This result is certainly contrary to my expectations, especially as the force required to make the contacts secure is considerable. The trough containing the bar *S*, when filled with melting ice, weighs about twenty-five pounds, and this entire mass is required to be moved by the pressure of the stop attached to the microscope carriage. I conclude, therefore, that with ordinary care there is no danger of wear with steel surfaces which have received a temper as high as possible.

But in order te provide against this supposed source of error, the high and low temperature observations were made in close proximity.

After the observations for the absolute coefficients of the bars *T*, *C. S.*, and R_2 were completed, these bars, with R_1, were removed to the comparing-room in Harvard Hall. The series of observations for the relative coefficients between these bars extends from March 22 to June 28, 1883, but the series from which the equations of condition were formed terminated on June 7. A few subsequent observations were made as a check upon the results obtained from the equations.

The observations and reductions, both for the absolute and the relative coefficients, are given on the following pages.

COMPARISON OF LINE-METER T WITH END-METER S IN MELTING ICE WITH ONE-INCH OBJECTIVE.

(1 div. = 504 μ.)

SERIES I.

Date.	Y 61	$S - T^{b_1}$	Date.	Y 61	$S - T^{b_1}$
1883.	°	div.	1883.	°	div.
Feb. 7	+ 1.20	+ 25.7	Feb. 14	— 6.64	+278.4
"	+ 1.70	+ 30.4	"	— 7.18	+293.5
"	+ 1.70	+ 20.1	"	— 6.18	+270.3
"	+ 1.70	+ 24.5	"	— 6.03	+259.8
"	+ 2.60	— 12.5	"	— 6.04	+263.4
"	+ 4.20	— 60.0	"	— 7.60	+308.6
"	+ 3.40	— 38.7	"	— 7.56	+310.9
"	+ 3.40	— 34.5	"	— 6.06	+260.0
"	+ 4.50	— 76.0	"	— 6.66	+280.6
"	+ 7.65	—172.4			
			Feb. 15	+ 5.83	—118.4
Feb. 8	— 4.40	+213.8	"	+ 5.88	—116.9
"	— 4.30	+216.5	"	+ 5.90	—113.1
"	— 4.30	+219.4	"	+ 5.88	—120.6
"	— 4.50	+218.9	"	+ 5.90	—115.4
"	— 4.60	+220.4			
"	— 4.40	+217.7	Feb. 16	— 0.28	+ 75.6
"	— 4.40	+219.3	"	— 0.20	+ 82.1
"	+ 1.50	+ 18.6	"	+ 0.70	+ 55.8
"	+ 1.50	+ 15.5	"	+ 1.88	+ 17.9
"	+ 1.40	+ 16.4	"	+ 1.90	+ 16.7
"	+ 1.50	+ 19.2			
			Feb. 18	+28.78	—862.6
Feb. 9	— 4.70	+222.3	"	+28.80	—850.3
"	— 4.70	+228.0	"	+28.72	—863.7
"	— 4.80	+218.8	"	+28.44	—852.3
"	— 4.80	+218.9	"	+28.24	—847.4
"	— 4.70	+219.2	"	+28.58	—844.8
"	— 4.90	+221.3	"	+28.20	—837.2
			"	+28.56	—846.0
Feb. 11	+13.02	—342.6	"	+28.38	—838.3
"	+13.02	—343.5			
"	+13.92	—379.1	Feb. 19	+24.54	—720.2
"	+13.82	—375.1	"	+24.52	—726.4
			"	+24.24	—712.5
Feb. 12	+ 1.80	+ 20.4	"	+24.24	—704.0
"	— 2.94	+166.7	"	+24.24	—708.0
"	— 3.16	+171.3	"	+24.57	—719.7
"	— 0.48	+ 94.4	"	+24.57	—729.6
"	— 0.22	+ 81.3			
"	— 1.10	+ 98.4	Feb. 20	— 2.21	+142.8
"	— 1.48	+119.2	"	— 2.21	+139.9
			"	— 1.77	+128.4
Feb. 13	+ 2.38	+ 7.2	"	— 1.75	+120.6
"	+ 1.38	+ 26.2			
"	+ 1.38	+ 30.8	Feb. 25	+ 2.46	— 2.5
"	— 0.06	+ 79.0	"	+ 2.46	— 4.9
"	+ 0.04	+ 71.2	"	+ 3.48	— 32.9
"	+ 0.37	+ 61.9	"	+ 3.28	— 26.5
"	+ 3.30	— 26.8	"	+ 3.30	— 27.7
"	+ 3.30	— 26.1	"	+ 6.43	—129.8

Series I. — Continued.

Date.	Y 61	$S-T^{b_1}$	Date.	Y 61	$S-T^{b_1}$
1883.	°	div.	1883.	°	div.
Feb. 25	+ 7.80	—176.7	Feb. 27	— 9.96	+386.8
"	+ 7.80	—175.6	"	— 9.96	+379.0
"	+ 8.06	—174 1	"	— 6.87	+294.9
"	+ 8.06	—179.5	"	— 6.87	+291.1
"	+ 8.24	—182.2	"	— 6.88	+295.6
"	+ 8.24	—182.2	"	+ 2.64	— 12.8
"	+ 8.26	—191.2	"	+ 2.74	— 12.2
"	+ 8.64	—196.5	"	+ 2.80	— 10.5
"	+ 8.30	—188.4	"	+ 3.18	— 25.6
"	+ 8.34	—182.2	"	+ 3.20	— 23.7
"	+ 8 34	—182.8	"	+ 3.12	— 28.2
"	+ 8.46	—189.4			
			Feb. 28	— 3.73	+193.1
Feb. 26	+22.17	—642.5	"	— 3.73	+193.4
"	+21.95	—628.6	"	— 3.66	+192.0
"	+21.88	—635.4	"	— 3.66	+192.2
"	+21.86	—635.0	"	— 3.34	+185.1
"	+22.74	—658.4	"	— 3.34	+183.0
"	+22.72	—657.4	"	— 0.18	+ 80.1
"	+22.62	—659.1	"	— 0.18	+ 85.7
			"	— 0.28	+ 83.4
Feb. 27	— 9.82	+376.0	"	— 0.30	+ 86.3

Series II.

Date.	Time.	Y 61	$S-T^{b_1}$	Date.	Time.	Y 61	$S-T^{b_1}$
1883.	h. m.	°	div.	1883.	h. m.	°	div.
Feb. 28	6 0 A.M.	— 0.78	+ 77.3	Mar. 4	6 3 A.M.	— 0.14	+ 63.9
"	6 5 "	— 0.78	+ 91.5	"	6 20 "	— 0.28	+ 70.4
"	6 12 "	— 0.68	+ 93.9	"	6 24 "	— 0.30	+ 69.2
"	6 15 "	— 0.68	+ 91.6	"	6 40 "	— 0.52	+ 80.2
"	6 18 "	— 0.68	+ 84.8	"	6 43 "	— 0.52	+ 79.2
"	6 20 "	— 0.68	+ 82.4	"	7 0 "	— 1.18	+ 90.2
"	6 30 "	— 0.66	+ 81.8	"	7 3 "	— 1.15	+ 90.8
"	6 35 "	— 0.66	+ 87.0	"	8 0 "	+ 0.34	+ 50.5
				"	8 3 "	+ 0.34	+ 45.8
Mar. 1	3 30 P.M.	+ 3.64	— 56.5				
"	3 33 "	+ 3.64	— 52.3	Mar. 5	6 0 A.M.	— 5.08	+232.0
"	4 30 "	+ 3.68	— 57.6	"	6 5 "	— 5.08	+219.7
"	4 40 "	+ 3.72	— 54.8	"	6 30 "	— 3.92	+191.2
"	4 45 "	+ 3.72	— 53.0	"	6 35 "	— 3.94	+186.8
				"	7 0 "	— 1.26	+106.2
Mar. 3	8 25 A.M.	+19.26	—560.5	"	7 3 "	— 1.26	+ 99.3
"	8 30 "	+19 26	—561.1	"	7 10 "	— 1.22	+103.5
"	8 40 "	+19.28	—565.4	"	7 13 "	— 1.22	+101.2
"	8 50 "	+19.28	—562.5	"	7 30 "	— 1.18	+107.1
				"	7 33 "	— 1.18	+103.5
Mar. 4	7 20 A.M.	—10.34	+386.0				
"	7 25 "	—10.37	+386.4	Mar. 6	5 45 A.M.	— 7.98	+316.3
"	7 30 "	—10.45	+383.7	"	5 48 "	— 7.98	+304.7
"	7 50 "	—10.44	+386.5	"	6 0 "	— 7.75	+299.7
"	7 55 "	—10.40	+382.2	"	6 3 "	— 7.75	+299.7
"	6 0 P.M.	— 0.14	+ 63.8	"	6 12 "	— 7.77	+303.9

SERIES II. — Continued.

Date.	Time.	Y 61	$S - T^{b_1}$	Date.	Time.	Y 61	$S - T^{b_1}$
1883.	h. m.	°	div.	1883.	h. m.	°	div.
Mar. 6	6 15 A.M.	— 7.77	+294.3	Apr. 24	7 55 A.M.	+ 5.60	—118.5
"	6 30 "	— 7.79	+308.5	"	7 58 "	+ 5.65	—122.5
"	6 33 "	— 7.79	+304.5	"	8 0 "	+ 5.72	—120.8
"	7 0 "	— 7.90	+305.9	"	8 2 "	+ 5.72	—124.1
"	7 3 "	— 7.90	+308.4	"	8 5 "	+ 5.82	—121.7
"	7 20 "	— 8.34	+327.8	"	8 7 "	+ 5.85	—126.5
"	7 23 "	— 8.34	+324.5	"	8 12 "	+ 5.90	—125.7
				"	8 15 "	+ 5.98	—129.0
Apr. 15	7 0 A.M.	+19.47	—563.3	"	2 35 P.M.	+25.18	—748.4
"	7 30 "	+19.47	—567.7	"	2 50 "	+25.26	—750.5
"	7 30 "	+19.54	—565.1	"	3 5 "	+25.26	—750.4
"	8 10 "	+18.94	—547.1	"	3 7 "	+25.26	—749.5
				"	3 9 "	+25.30	—749.6
Apr. 17	7 0 A.M.	+12.24	—334.5	"	3 11 "	+25.30	—750.6
"	7 10 "	+12.22	—334.3	"	3 20 "	+25.28	—754.2
"	7 50 "	+12.44	—330.3	"	3 28 "	+25.28	—751.2
"	8 10 "	+12.06	—331.6	"	3 30 "	+25.28	—759.7
				"	3 40 "	+25.28	—756.7
Apr. 18	6 20 A.M.	+ 8.47	—210.2	"	3 45 "	+25.26	—753.8
"	6 23 "	+ 8.34	—200.0	"	3 50 "	+25.32	—753.7
"	6 27 "	+ 8.61	—219.4	"	4 0 "	+25.30	—750.3
"	6 30 "	+ 8.74	—215.4	"	4 3 "	+25.30	—749.5
"	6 45 "	+ 9.16	—230.9	"	8 50 "	+20.54	—604.0
"	7 10 "	+11.54	—307.7	"	8 55 "	+20.54	—598.9
				"	9 0 "	+20.42	—593.4
Apr. 19	8 0 A.M.	+ 7.66	—186.6	"	9 5 "	+20.40	—590.9
"	8 13 "	+ 7.84	—187.9	"	9 10 "	+20.36	—588.9
"	9 0 "	+ 9.70	—248.6	"	9 12 "	+20.28	—590.9
"	9 13 "	+ 9.48	—241.9	"	9 15 "	+20.28	—590.0
Apr. 20	8 0 A.M.	+17.88	—513.9	Apr. 25	6 30 P.M.	+ 2.46	— 19.2
"	12 0 "	+17.67	—504.9	"	6 33 "	+ 2.46	— 20.5
"	4 0 P.M.	+18.22	—531.5	"	6 35 "	+ 2.44	— 18.1
				"	6 38 "	+ 2.44	— 19.5
Apr. 22	5 40 P.M.	+ 4.24	— 77.4	"	6 45 "	+ 2.48	— 18.6
"	5 43 "	+ 4.24	— 76.3	"	6 48 "	+ 2.52	— 20.9
"	7 10 "	+ 4.94	— 94.9	"	6 55 "	+ 2.55	— 22.6
"	7 40 "	+ 5.10	— 99.0	"	7 5 "	+ 2.62	— 25.3
				"	8 20 A.M.	+ 3.85	— 61.6
Apr. 23	8 0 A.M.	+ 5.97	—130.6	"	8 25 "	+ 3.85	— 60.1
"	8 3 "	+ 5.97	—131.6	"	8 38 "	+ 3.85	— 62.8
"	8 20 "	+ 6.14	—135.2	"	8 42 "	+ 3.85	— 63.2
"	8 23 "	+ 6.14	—132.3	"	8 48 "	+ 3.70	— 56.4
"	8 37 "	+ 5.97	—127.8	"	8 50 "	+ 3.60	— 56.3
"	8 40 "	+ 5.97	—127.0	"	8 55 "	+ 3.70	— 62.1
				"	9 5 "	+ 4.00	— 64.5
Apr. 24	6 25 A.M.	+ 5.88	—128.7	"	2 45 P.M.	+19.80	—582.4
"	6 28 "	+ 5.95	—132.8	"	2 55 "	+19.74	—584.8
"	6 33 "	+ 6.05	—133.7	"	3 5 "	+19.76	—584.7
"	6 35 "	+ 6.10	—136.3	"	3 7 "	+19.76	—583.4
"	6 40 "	+ 6.16	—132.5	"	3 30 "	+20.10	—592.1
"	6 43 "	+ 6.20	—135.5	"	3 35 "	+20.04	—588.9
"	6 50 "	+ 6.22	—137.9	"	3 37 "	+20.04	—588.2
"	7 40 "	+ 5.52	—115.4	"	3 47 "	+20.05	—586.6
"	7 44 "	+ 5.52	—115.9	"	3 49 "	+20.03	—585.7

SERIES II. — Continued.

Date.	Time.	Y 61	$S - T^{b_1}$	Date.	Time.	Y 61	$S - T^{b_1}$
1883.	h. m.	°	div.	1883.	h. m.	°	div.
Apr. 25	3 55 P.M.	+20.10	—587.8	Apr. 29	5 17 P.M.	+27.32	—822.6
"	3 57 "	+20.10	—590.0	"	5 23 "	+27.28	—819.2
				"	5 32 "	+27.44	—827.1
Apr. 26	7 10 A.M	+ 2.24	— 13.0	"	5 45 "	+27.62	—833.5
"	7 15 "	+ 2.24	— 16.7	"	5 55 "	+27.62	—832.6
"	7 20 "	+ 2.24	— 11.0	"	6 23 "	+27.46	—825.6
"	7 23 "	+ 2.24	— 11.3				
"	7 29 "	+ 2.30	— 16.7	Apr. 30	6 45 A.M.	+ 3.28	— 43.4
"	7 33 "	+ 2.30	— 16.4	"	6 50 "	+ 3.48	— 52.5
"	12 5 P.M.	+21.04	—627.6	"	7 0 "	+ 4.06	— 63.8
"	12 15 "	+21.22	—623.0	"	1 0 P.M.	+15.88	—453.5
"	12 18 "	+21.22	—617.7	"	1 3 "	+15.88	—454.0
"	12 23 "	+21.12	—629.3	"	1 10 "	+15.86	—452.0
"	1 20 "	+21.23	—619.4	"	1 13 "	+15.86	—448.9
"	1 23 "	+21.23	—617.5	"	1 17 "	+15.80	—443.7
"	4 10 "	+25.90	—776.4	"	1 19 "	+15.80	—441.8
"	4 12 "	+26.07	—783.3	"	6 30 "	+18.02	—524.4
"	4 20 "	+25.86	—777.9	"	6 33 "	+18.02	—519.3
"	4 25 "	+25.86	—767.8	"	7 0 "	+17.74	—510.2
"	4 35 "	+26.00	—772.8	"	7 0 "	+17.74	—507.9
"	4 45 "	+26.10	—784.7	"	7 15 "	+17.50	—502.0
"	5 0 "	+25.84	—780.8	"	7 18 "	+17.50	—503.4
"	5 5 "	+25.84	—769.5	"	7 30 "	+17.60	—507.8
				"	7 33 "	+17.60	—506.1
Apr. 27	6 40 A.M.	+ 5.44	—107.2				
"	6 42 "	+ 5.44	—110.0	May 1	6 0 A.M.	+ 5.41	—112.1
"	6 46 "	+ 5.66	—118.5	"	6 3 "	+ 5.41	—116.9
"	6 48 "	+ 5.66	—121.1	"	6 10 "	+ 5.50	—117.9
"	6 53 "	+ 5.86	—124.9	"	6 13 "	+ 5.58	—120.2
"	6 56 "	+ 5.86	—122.9	"	6 20 "	+ 5.82	—127.5
"	8 55 "	+ 8.84	—223.5	"	6 24 "	+ 5.82	—130.5
"	8 57 "	+ 8.97	—227.8	"	6 40 "	+ 6.26	—132.8
"	9 5 "	+ 9.15	—234.4	"	6 55 "	+ 6.48	—145.3
"	9 8 "	+ 9.20	—232.5	"	6 58 "	+ 6.50	—146.1
"	1 10 P.M.	+14.86	—416.4	"	12 55 P.M.	+17.63	—507.9
"	2 5 "	+16.42	—468.2	"	1 0 "	+17.63	—504.4
"	2 7 "	+16.42	—469.3	"	1 10 "	+17.66	—503.5
"	2 15 "	+16.64	—475.8	"	1 13 "	+17.66	—504.2
"	2 17 "	+16.64	—472.9	"	7 20 "	+16.50	—473.9
"	2 30 "	+16.70	—478.8	"	7 23 "	+16.50	—475.3
"	2 33 "	+16.70	—478.3	"	7 30 "	+16.56	—469.9
				"	7 33 "	+16.56	—472.6
Apr. 28	7 35 A.M.	+18.48	—534.2				
"	7 38 "	+18.48	—534.0	May 2	7 0 A.M.	+ 8.70	—218.0
				"	7 5 "	+ 8.80	—220.6
Apr. 29	7 0 A.M.	+15.33	—431.1				
"	7 3 "	+15.33	—430.2	May 4	6 25 A.M.	+18.12	—518.5
"	8 15 "	+15.86	—449.7	"	6 28 "	+18.12	—513.7
"	8 17 "	+15.86	—449.4	"	6 30 "	+18.00	—513.5
"	8 37 "	+15.98	—455.1	"	6 40 "	+17.93	—508.8
"	8 40 "	+15.98	—454.4	"	6 43 "	+17.93	—507.6
"	8 55 "	+16.04	—457.8	"	8 20 "	+18.04	—518.8
"	8 58 "	+16.04	—458.2	"	8 23 "	+18.04	—518.0
"	9 3 "	+16.04	—452.5	"	8 30 "	+18.10	—514.6
"	9 5 "	+16.04	—454.2	"	8 33 "	+18.10	—515.1

Series II. — Continued.

Date.	Time.	Y 61	$S - T^{b_1}$	Date.	Time.	Y 61	$S - T^{b_1}$
1883.	h. m.	°	div.	1883.	h. m.	°	div.
May 4	8 40 P.M.	+18.18	—521.0	May 7	5 34 A.M.	+ 8.00	—193.3
"	8 43 "	+18.18	—519.7	"	6 50 "	+29.25	—884.4
"	2 55 P.M.	+20.05	—582.8	"	6 53 "	+29.25	—879.7
"	2 59 "	+20.05	—580.8	"	7 0 "	+29.14	—879.3
				"	7 3 "	+29.14	—876.3
May 6	9 0 A.M.	+16.44	—465.6				
"	9 3 "	+16.44	—466.1				
				May 8	5 25 A.M.	+12.82	—359.3
May 7	5 20 A.M.	+ 7.90	—197.3	"	5 30 "	+12.82	—347.8
"	5 24 "	+ 7.95	—197.8	"	5 35 "	+12.82	—349.2
"	5 30 "	+ 7.98	—192.4	"	5 40 "	+12.82	—348.7

EQUATIONS OF CONDITION BETWEEN T AND S.

Series I.

$$S - T^{b_1} = a + (\tau - 0^{\circ})\, b$$

1883.	$S - T^{b_1}$		$(\tau - 0^{\circ})$	a	Δa	Δa
Feb. 7	+ 17.6 div.	=	a + 1.76 b	+74.5	—0.5 div.	—0.2 μ
" 7	— 52.3 div.	=	a + 3.83 b	+71.6	—3.4 div.	—1.7 μ
" 7	—172.4 div.	=	a + 7.60 b	+73.4	—1.6 div.	—0.8 μ
" 8	+218.0 div.	=	a — 4.30 b	+78.9	+3.9 div.	+1.9 μ
" 8	+ 20.4 div.	=	a + 1.46 b	+67.6	—7.4 div.	—3.7 μ
" 9	+221.4 div.	=	a — 4.54 b	+75.1	+0.1 div.	+0.0 μ
" 11	—360.1 div.	=	a +13.47 b	+75.4	+0.4 div.	+0.2 μ
" 12	+ 20.4 div.	=	a + 1.78 b	+77.9	+2.9 div.	+1.4 μ
" 12	+169.0 div.	=	a — 2.96 b	+73.3	—1.7 div.	—0.8 μ
" 12	+ 98.3 div.	=	a — 0.79 b	+72.8	—2.2 div.	—1.1 μ
" 13	+ 21.4 div.	=	a + 1.69 b	+76.0	—1.0 div.	—0.5 μ
" 13	+ 70.7 div.	=	a + 0.12 b	+74.6	—0.4 div.	—0.2 μ
" 13	— 26.4 div.	=	a + 3.26 b	+79.0	+4.0 div.	+2.0 μ
" 14	+280.6 div.	=	a — 6.50 b	+70.4	—4.6 div.	—2.3 μ
" 15	—116.9 div.	=	a + 5.82 b	+71.3	—3.7 div.	—1.8 μ
" 16	+ 49.6 div.	=	a + 0.78 b	+74.8	—0.2 div.	—0.1 μ
" 18	—849.2 div.	=	a +28.40 b	+69.1	—5.9 div.	—2.9 μ
" 19	—719.5 div.	=	a +24.53 b	+73.8	—1.2 div.	—0.6 μ
" 20	+132.9 div.	=	a — 1.92 b	+70.8	—4.2 div.	—2 1 μ
" 25	— 18.9 div.	=	a + 2.96 b	+76.8	+1.8 div.	+0.9 μ
" 25	—167.1 div.	=	a + 7.58 b	+78.0	+3.0 div.	+1.5 μ
" 25	—186.8 div.	=	a + 8.31 b	+81.9	+6.9 div.	+3.4 μ
" 26	—645.2 div.	=	a +22.37 b	+78.0	+3.0 div.	+1.5 μ
" 27	+380.6 div.	=	a — 9.59 b	+70.5	—4.5 div.	—2.3 μ
" 27	+293.9 div.	=	a — 6.71 b	+76.9	+1.9 div.	+0.9 μ

1883.	$S - T^{b_1}$		$(\tau - 0^\circ)$	a	Δa	Δa
Feb. 27	— 18.8 div.	=	$a + 2.91\,b$	+75.3	+0.3 div.	+0.1 μ
" 28	+189.8 div.	=	$a - 3.47\,b$	+77.6	+2.6 div.	+1.3 μ
" 28	+ 83.9 div.	=	$a - 0.22\,b$	+76.8	+1.8 div.	—0.9 μ

Normal Equations.

$$-1065.1 = 28\,a + 97.63\,b \qquad b = -32.34$$
$$-76975.0 = +97.63\,a + 260.67\,b \qquad a = +75.0$$

EQUATIONS OF CONDITION BETWEEN T AND S.

Series II.

1883.	$S - T^{b_1}$		$(\tau - 0^\circ)$	a	Δa	Δa
Feb. 28	+ 86.3 div.	=	$a - 0.71\,b$	+63.5	+2.2 div.	+1.1 μ
Mar. 1	— 54.8 div.	=	$a + 3.64\,b$	+62.1	+0.8 div.	+0.4 μ
" 3	—562.4 div.	=	$a + 19.33\,b$	+58.5	—2.5 div.	—1.3 μ
" 4	+385.0 div.	=	$a - 10.05\,b$	+62.2	+0.9 div.	+0.4 μ
" 4	+ 70.4 div.	=	$a - 0.35\,b$	+59.2	—2.1 div.	—1.0 μ
" 5	+207.4 div.	=	$a - 4.38\,b$	+66.7	+5.4 div.	+2.7 μ
" 5	+103.5 div.	=	$a - 1.18\,b$	+65.6	+4.3 div.	+2.2 μ
" 6	+308.2 div.	=	$a - 7.72\,b$	+60.3	—1.0 div.	—0.5 μ
April 15	—560.8 div.	=	$a + 19.41\,b$	+62.7	+1.4 div.	+0.7 μ
" 17	—332.7 div.	=	$a + 12.25\,b$	+60.8	—0.5 div.	—0.3 μ
" 18	—230.6 div.	=	$a + 9.14\,b$	+63.0	+1.7 div.	+0.8 μ
" 19	—216.2 div.	=	$a + 8.64\,b$	+61.3	+0.0 div.	+0.0 μ
" 20	—516.8 div.	=	$a + 17.96\,b$	+60.1	—1.2 div.	—0.6 μ
" 22	— 86.9 div.	=	$a + 4.57\,b$	+59.9	—1.4 div.	—0.7 μ
" 23	—130.8 div.	=	$a + 5.97\,b$	+60.9	—0.4 div.	—0.2 μ
" 24	—133.9 div.	=	$a + 6.02\,b$	+59.5	—1.8 div.	—0.9 μ
" 24	—122 0 div.	=	$a + 5.67\,b$	+60.1	—1.2 div.	—0.6 μ
" 24	—752.0 div.	=	$a + 25.40\,b$	+64.0	+2.7 div.	+1.3 μ
" 24	—593.9 div.	=	$a + 20.47\,b$	+63.6	+2.3 div.	+1.2 μ
" 25	— 20.6 div.	=	$a + 2.47\,b$	+58.7	—2.6 div.	—1.3 μ
" 25	— 60.9 div.	=	$a + 3.75\,b$	+59.5	—1.8 div.	—0.9 μ
" 25	—586.8 div.	=	$a + 20.04\,b$	+57.0	—4.3 div.	—2.1 μ
" 26	— 14.2 div.	=	$a + 2.24\,b$	+57.8	—3.5 div.	—1.7 μ
" 26	—622.4 div.	=	$a + 21.26\,b$	+60.6	—0.7 div.	—0.4 μ
" 26	—776.7 div.	=	$a + 26.07\,b$	+60.7	—0.6 div.	—0.3 μ
" 27	—117.4 div.	=	$a + 5.59\,b$	+62.2	+0.9 div.	+0.5 μ
" 27	—229.6 div.	=	$a + 9.02\,b$	+60.1	—1.2 div.	—0.6 μ
" 27	—465.7 div.	=	$a + 16.37\,b$	+60 1	—1.2 div.	—0.6 μ
" 28	—534.1 div.	=	$a + 18.54\,b$	+61.5	+0.2 div.	+0.1 μ
" 29	—430.6 div.	=	$a + 15.35\,b$	+62.5	+1.2 div.	+0.6 μ

1883.	$S - T^{b_1}$			$(\tau - 0°)$	a	Δa	Δa
April 29	—453.9 div.	=	a	+16.01 b	+60.4	—0.9 div.	—0.4 μ
" 29	—826.8 div.	=	a	+27.60 b	+59.7	—1.6 div.	—0.8 μ
" 30	— 53.2 div.	=	a	+ 3.57 b	+61.5	+0.2 div.	+0.1 μ
" 30	—449.0 div.	=	a	+15.88 b	+61.2	—0.1 div.	+0.0 μ
" 30	—510.1 div.	=	a	+17.76 b	+60.4	—0.9 div.	—0.4 μ
May 1	—127.7 div.	=	a	+ 5.81 b	+58.9	—2.4 div.	—1.2 μ
" 1	—505.0 div.	=	a	+17.68 b	+63.0	+1.7 div.	+0.9 μ
" 1	—472.9 div.	=	a	+16.56 b	+59.1	—2.2 div.	—1.1 μ
" 2	—219.3 div.	=	a	+ 8.72 b	+60.8	—0.5 div.	—0.3 μ
" 4	—512.4 div.	=	a	+18.07 b	+68.0	+6.7 div.	+3.3 μ
" 4	—517.8 div.	=	a	+18.16 b	+65.6	+4.3 div.	+2.2 μ
" 4	—581.8 div.	=	a	+20.12 b	+64.5	+3.2 div.	+1.6 μ
" 6	—465.8 div.	=	a	+16.47 b	+63.3	+2.0 div.	+2.0 μ
" 7	—195.2 div.	=	a	+ 7.92 b	+59.2	—2.1 div.	—1.0 μ
" 7	—879.9 div.	=	a	+29.34 b	+62.5	+1.2 div.	+0.6 μ
" 8	—351.3 div.	=	a	+12.84 b	+61.2	—0.1 div.	—0.1 μ

Normal Equations.

$$-14114.1 = 46\,a + 527.29\,b \qquad b = -32.12$$
$$-288794.2 = 527.29\,a + 9999.52\,b \qquad a = +61.3$$

Combination of Results.

		Weight.
Series I.	$b = -32.34$ div.	1
Series II.	$b = -32.12$ div.	3

Hence:—

$b = -32.18$ div. $= -16.22\,\mu$ for each degree Centigrade.

COMPARISON OF LINE-METER *C. S.* WITH END-METER *S* IN MELTING ICE WITH ONE-INCH OBJECTIVE.

(1 div. = .504 μ.)

Series I.

Date.	Y 61	*S— C.S.*	Date.	Y 61	*S— C.S.*
1883.	°	div.	1883.	°	div.
Feb. 11	+13.00	+ 42.7	Feb. 13	+ 4.77	+339.5
"	+13.00	+ 54.0	"	+ 4.75	+341.5
"	+13.00	+ 61.2	"	+ 4.72	+341.4
"	+13.00	+ 55.2	"	— 7.92	+778.0
			"	— 7.92	+769.0
Feb. 12	— 0.14	+501.2	"	— 7.98	+767.8
"	— 0.14	+503.3	"	— 7.94	+775.5
"	— 1.28	+541.1			
"	— 1.16	+533.6	Feb. 14	— 4.98	+664.3
"	— 1.16	+545.9	"	— 5.02	+683.6
"	— 1.06	+544.3			
"	— 1.28	+549.4	Feb. 15	+ 6.42	+283.9

Series I. — Continued.

Date.	Y 61	$S-C.S.$	Date.	Y 61	$S-C.S.$
1883.	°	div.	1883.	°	div.
Feb. 15	+ 6.42	+274.8	Feb. 25	+ 8.28	+220.8
"	+ 6.26	+280.2	"	+ 8.30	+215.9
"	+ 6.26	+275.3	"	+ 8.34	+218.5
"	+ 5.92	+283.8	"	+ 8.34	+215.5
"	+ 5.95	+291.9	"	+ 8.34	+218.7
Feb. 16	— 1.46	+537.1	Feb. 26	+22.96	—292.9
"	— 1.48	+539.4	"	+22.94	—295.4
"	— 1.50	+536.2	"	+23.07	—305.3
"	— 1.49	+536.6	"	+23.07	—299.0
"	— 1.48	+540.6	"	+21.88	—268.1
"	— 1.48	+541.7	"	+21.86	—270.0
"	— 1.48	+542.7	"	+21.86	—268.6
Feb. 18	+27.45	—464.4	Feb. 27	+ 0.55	+468.1
"	+27.36	—467.9	"	+ 0.60	+467.0
"	+27.48	—470.5	"	+ 0.70	+464.8
"	+27.66	—461 8	"	+ 0.80	+462.3
"	+27.66	—471.2	"	+ 0.74	+468.6
"	+27.76	—472.8	"	—10.10	+835.4
			"	—10.08	+835.2
Feb. 19	+27.22	—456.1	"	—10.14	+837.5
			"	—10.14	+842.5
Feb. 20	— 3 11	+590.1			
"	— 2.99	+596 4	Feb. 28	— 2 66	+594.3
			"	— 2.66	+590.9
Feb. 25	+ 1.74	+439.1	"	— 1.10	+542.8
"	+ 1.76	+443.8	"	— 1.10	+534.8
"	+ 1.78	+448.1	"	— 0.11	+507.7
"	+ 1.80	+443.4	"	— 0.11	+502.2

Series II.

Date.	Time.	Y 61	$S-C.S.$	Date.	Time.	Y 61	$S-C.S$
1883.	h. m.	°	div.	1883.	h. m	°	div.
Feb. 28	7 3 A.M	— 0.84	+513.4	Mar. 1	10 30 A.M.	+ 5.97	+272.0
"	7 9 "	— 0.84	+517.6	"	10 35 "	+ 5 98	+274.6
"	7 12 "	— 0.79	+514.5	"	11 0 "	+ 5.44	+298.7
"	7 15 "	— 0.81	+511.7	"	11 10 "	+ 5.44	+298.5
"	8 0 "	— 2.18	+558.8				
"	8 3 "	— 2.18	+555.4	Mar. 3	7 10 A.M.	+19.24	—192.4
"	8 15 "	— 2.60	+577.0	"	7 30 "	+19.22	—190.8
"	8 20 "	— 2.64	+573.0	"	7 40 "	+19.15	—178.9
"	10 5 "	— 3.86	+616.9	"	7 55 "	+19.15	—181.0
"	10 8 "	— 3.88	+612.6				
"	3 20 P.M.	+ 3.14	+381 1	Mar. 4	7 50 A.M.	—10.60	+836.2
"	3 25 "	+ 3.14	+374.5	"	7 55 "	—10.57	+836.3
"	3 40 "	+ 3.30	+382.7	"	8 0 "	—10.48	+834.4
"	3 45 "	+ 3.12	+378 6	"	8 5 "	—10.45	+833 0
"	3 50 "	+ 3.16	+376.9	"	8 15 "	—10.40	+834.7
"	3 55 "	+ 3.16	+376.2	"	8 20 "	—10.40	+832.1

SERIES II. — Continued.

Date.	Time.	Y 61	S—C.S.	Date.	Time.	Y 61	S—C.S.
1883.	h. m.	°	div.	1883.	h. m.	°	div.
Mar. 4	8 30 A.M.	—10.10	+829.5	Apr. 22	7 30 A.M.	+ 4.36	+337.5
"	8 35 "	—10.08	+825.3	"	6 0 P.M.	+24.28	—371.1
"	8 20 "	— 9.34	+807.9	"	6 20 "	+24.40	—370.1
"	8 25 "	— 9.30	+802.7	"	6 30 "	+24.40	—367.0
"	9 0 "	— 9.68	+816.4	"	6 40 "	+24.40	—365.0
"	9 10 "	— 9.64	+816.4				
"	9 30 "	— 9.11	+798.1	Apr. 23	6 30 A.M.	+ 5.88	—284.1
"	9 40 "	— 9.09	+794.9	"	6 35 "	+ 5.88	—282.7
"	10 25 "	— 8 34	+768.8	"	6 45 "	+ 5.90	—287.9
"	10 30 "	— 8.20	+760.4	"	6 50 "	+ 5.90	—287.3
"	12 30 P.M.	— 4.56	+648.9	"	6 55 "	+ 5.90	—284.8
"	12 33 "	— 4.50	+643.2	"	7 0 "	+ 5.88	—281.3
"	4 40 "	— 0.92	+524.6	"	7 10 "	+ 5.74	—286.0
"	4 43 "	— 0.90	+519.4	"	7 12 "	+ 5.74	—282.1
"	5 10 "	— 0.74	+507.6	"	8 45 "	+ 5 88	—282.0
"	5 13 "	— 0.78	+509.6	"	8 50 "	+ 5.88	—281.3
"	5 18 "	— 0.76	+511.3	"	8 55 "	+ 5.88	—286.9
"	5 20 "	— 0.94	+517.1	"	9 0 "	+ 5.88	—281.5
"	5 22 "	— 0.96	+511.7				
"	5 25 "	— 0.76	+510.0	Apr. 24	5 40 A.M.	+ 5.90	+284.9
"	5 30 "	— 1.10	+520.1	"	5 45 "	+ 5.84	+288.4
"	5 33 "	— 1.16	+519.5	"	5 48 "	+ 5.84	+286.0
"	5 40 "	— 1.00	+518.7	"	6 0 "	+ 5.78	+281.8
"	5 43 "	— 1.00	+518.3	"	6 3 "	+ 5.78	+281.7
				"	6 5 "	+ 5.84	+287.9
Mar. 5	7 20 A.M.	—12 80	+911.8	"	6 8 "	+ 5.84	+286.7
"	7 23 "	—12.80	+902.3	"	6 15 "	+ 5.84	+285.8
"	8 15 "	—12.70	+916.8	"	6 18 "	+ 5.84	+284.9
"	8 20 "	—12.70	+908.1	"	9 25 "	+20.28	—216.7
				"	9 28 "	+20.20	—217.1
Apr. 18	6 0 A.M.	+ 8.87	+182.9				
"	6 5 "	+ 8.87	+179.9	Apr. 25	5 45 A.M.	+ 2.25	+408.5
				"	5 48 "	+ 2.25	+407.3
Apr. 19	7 0 A.M.	+ 7.04	+234.1	"	6 0 "	+ 2.25	+405.6
"	7 10 "	+ 7.00	+240.7	"	6 5 "	+ 2.25	+411.5
"	7 30 "	+ 6.62	+256.7	"	6 10 "	+ 2.25	+407.6
"	7 40 "	+ 6.42	+260.3	"	6 13 "	+ 2.25	+408.2
"	7 45 "	+ 6.56	+253.6	"	6 15 "	+ 2.25	+408.1
"	7 50 "	+ 6.84	+252.6	"	6 17 "	+ 2 25	+406.6
"	5 15 P.M.	+23 14	—323.5	"	4 25 P.M.	+20 32	—216.3
"	5 20 "	+23.00	—313.6	"	4 27 "	+20.32	—219.9
"	5 25 "	+22.90	—310.1	"	9 0 "	+20.74	—236.4
"	5 30 "	+22 70	—303 3	"	9 5 "	+20.74	—233.8
"	5 35 "	+22.54	—301.4				
"	5 45 "	+22.43	—290.2	Apr. 26	5 40 A.M.	+ 1.60	+425.3
				"	5 45 "	+ 1.60	+433.0
Apr. 20	6 20 A.M.	+17.74	—141.0	"	5 50 "	+ 1.50	+430.2
"	6 30 "	+17.74	—123.9	"	5 53 "	+ 1.50	+428.3
				"	5 55 "	+ 1.60	+427.4
Apr. 21	6 30 A.M.	+16.75	—100.9	"	6 0 "	+ 1.52	+427.3
				"	6 3 "	+ 1.52	+422.2
Apr. 22	7 0 A.M	+ 3 96	+339.5	"	6 18 "	+ 1.45	+424.6
"	7 3 "	+ 4.00	+345.3	"	6 22 "	+ 1.55	+420.8
"	7 10 "	+ 4.00	+344.5	"	6 25 "	+ 1.60	+423.5
"	7 28 "	+ 4.26	+336.6	"	6 35 "	+ 1.60	+423.7

SERIES II. — Continued.

Date.	Time.	Y 61	$S—C.S.$
1883.	h. m.	°	div.
Apr. 26	6 38 A.M	+ 1.65	+420.2
"	8 50 P.M.	+25.46	—395.1
"	8 53 "	+25.46	—394.0
"	8 57 "	+25.44	—399.0
"	8 59 "	+25.44	—396.0
Apr. 27	5 50 A.M.	+ 4.82	+322 5
"	5 55 "	+ 4.82	+322.5
"	6 5 "	+ 4.90	+322 0
"	6 8 "	+ 4.90	+317.3
"	6 12 "	+ 5.07	+314.4
"	6 15 "	+ 5.12	+311.0
"	6 25 "	+ 5.30	+294.2
"	6 28 "	+ 5.30	+300.8
"	4 0 P.M.	+25.45	—400.4
"	4 10 "	+25.45	—399.8
Apr. 28	7 50 A.M.	+18.66	—166.4
"	7 52 "	+18.66	—164.3
"	8 0 "	+18.66	—161.5
"	8 3 "	+18.66	—162.5
Apr. 29	1 55 P.M.	+ 8.05	+208.1
"	1 58 "	+ 8.05	+211.1
"	2 10 "	+ 8.12	+207.5
"	2 13 "	+ 8.12	+208.1
"	2 17 "	+ 8.12	+205.7
"	2 19 "	+ 8.12	+204.3
"	2 25 "	+ 8.20	+205.7
"	2 27 "	+ 8 20	+204.7
"	7 45 "	+27.26	—473.9
"	8 0 "	+27.16	—466.9
"	8 20 "	+27.00	—458.6
"	8 50 "	+27.16	—462.0
Apr 30	5 26 A.M.	+ 1.72	+413.9
"	5 29 "	+ 1.72	+406.8
"	5 44 "	+ 1.82	+403.6
"	5 48 "	+ 1.82	+400.0
"	5 55 "	+ 2.02	+396.2
"	6 0 "	+ 2 10	+402.9
"	6 10 "	+ 2.15	+398.8
"	6 20 "	+ 2.24	+392.5
May 1	5 15 A.M.	+ 5.13	+291.0
"	5 18 "	+ 5.13	+286.7
"	5 25 "	+ 5.13	+290.9
"	5 27 "	+ 5 13	+287.1
"	5 30 "	+ 5.20	+296.7
"	5 33 "	+ 5.20	+290.5
"	5 40 "	+ 5.26	+291.0
"	5 43 "	+ 5.26	+289.2
"	7 38 P.M.	+16.67	— 88.0
"	7 41 "	+16.67	— 89.5
"	7 50 "	+16.63	— 90.3
1883.	h. m.	°	div.
May 1	7 53 P.M.	+16.63	— 87.4
May 2	5 30 A.M.	+ 7.48	+216.9
"	5 33 "	+ 7.52	+211.4
"	5 36 "	+ 7.55	+216.3
"	5 39 "	+ 7.60	+214.3
"	5 40 "	+ 7.60	+214.8
"	5 48 "	+ 7.68	+214.6
"	5 51 "	+ 7.68	+213.0
"	12 50 P.M.	+18.02	—153.3
"	12 55 "	+18.02	—148.8
"	12 58 "	+18 02	—147.9
"	1 8 "	+18.07	—144.9
"	1 11 "	+18.07	—145.0
"	5 30 "	+18.85	—172.5
"	5 40 "	+18.85	—172.9
"	5 50 "	+18.93	—183.6
"	5 54 "	+18.98	—182.4
"	6 15 "	+19.04	—173.4
"	8 40 "	+19.06	—184.2
"	8 52 "	+19.06	—181.9
"	9 0 "	+19.12	—170.4
"	9 12 "	+19.12	—171.3
May 3	6 5 A.M.	+18.66	—171.4
"	6 8 "	+18.66	—170.1
"	6 15 "	+18 50	—162.8
"	6 17 "	+18 50	—165.8
"	6 25 "	+18.40	—157.5
"	6 28 "	+18.40	—161.0
"	6 45 "	+18.14	—152.2
"	6 48 "	+18.14	—149.9
"	8 30 "	+17.88	—137.4
"	8 33 "	+17.88	—137.4
"	8 40 "	+17.86	—137.0
"	8 43 "	+17.86	—137.5
"	1 0 P.M	+18 16	—143 3
"	1 3 "	+18.16	—143.8
"	1 15 "	+18.16	—139.6
"	1 18 "	+18.16	—141.8
May 4	5 40 A.M.	+18.46	—164.8
"	5 43 "	+18.46	—158.8
"	5 50 "	+18.35	—158.2
"	5 53 "	+18.35	—159.0
"	6 0 "	+18.25	—153 5
"	6 3 "	+18.25	—152.0
"	6 10 "	+18.16	—146.3
"	6 13 "	+18.16	—146.3
May 6	7 0 A.M.	+17.42	—124.1
"	7 3 "	+17.42	—123.0
"	9 12 "	+16.47	— 77.1
"	9 15 "	+16.47	— 78.2
"	9 45 "	+16.47	— 80.8

SERIES II. — Continued.

Date.	Time.	Y 61	$S-C.S.$	Date.	Time.	Y 61	$S-C.S.$
1883.	h. m.	°	div.	1883.	h. m.	°	div.
May 6	9 48 A.M.	+16.47	—84.6	May 7	5 45 A.M.	+ 7.80	+210.1
"	12 10 P.M.	+16.67	—93.6	"	5 48 "	+ 7.80	+213.7
"	12 13 "	+16.67	—94.9	"	6 10 "	+ 8.08	+200.0
"	12 20 "	+16.67	—91.1	"	6 12 "	+ 8.08	+198.8
"	12 23 "	+16.67	—91.2	"	6 35 P.M.	+29.64	—540.0
				"	6 37 "	+29.64	—544.1

EQUATIONS OF CONDITION BETWEEN METERS $C.S.$ AND $S.$

SERIES I.

1883.	$S-C.S$		$(\tau - 0°)$	a	Δa	Δa
Feb. 11	+ 53.3 div.	=	a +13.02 b	+500.1	+ 0.9 div.	+0.5 μ
" 12	+502.2 div.	=	a — 0.14 b	+497.3	— 1.9 div.	—1.0 μ
" 12	+542.9 div.	=	a — 1.15 b	+502.9	+ 3.7 div.	+1.8 μ
" 13	+340.8 div.	=	a + 4.69 b	+504.1	+ 4.9 div.	+2.4 μ
" 13	+772.6 div.	=	a — 7.76 b	+502.4	+ 3.2 div.	+1.6 μ
" 14	+673.9 div.	=	a — 4.87 b	+504.3	+ 5.1 div.	+2.6 μ
" 15	+281.6 div.	=	a + 6.14 b	+495.4	— 3.8 div.	—1.9 μ
" 16	+539.2 div.	=	a — 1.44 b	+487.8	—11.4 div.	—5.7 μ
" 18	—467.6 div.	=	a +27.56 b	+492.0	— 7.2 div.	—3.6 μ
" 18	—468.6 div.	=	a +27.84 b	+500.8	+ 1.6 div.	+0.8 μ
" 19	—456.1 div.	=	a +27.36 b	+496.6	— 2.6 div.	—1.3 μ
" 20	+593.3 div.	=	a — 2.96 b	+490.2	— 9.0 div.	—4.5 μ
" 25	+443.6 div.	=	a + 1.75 b	+504.5	+ 5.3 div.	+2.6 μ
" 25	+217.9 div.	=	a + 8.28 b	+506.1	+ 6.9 div.	+3.5 μ
" 26	—298.2 div.	=	a +23.11 b	+506.5	+ 7.3 div.	+3.6 μ
" 26	—268.9 div.	=	a +21.96 b	+495.7	— 3.5 div.	—1.7 μ
" 27	+466.2 div.	=	a + 0.67 b	+489.5	— 9.7 div.	—4.8 μ
" 27	+837.6 div.	=	a — 9.79 b	+496.7	— 2.5 div.	—1.3 μ
" 28	+592.6 div.	=	a — 2.58 b	+502.8	+ 3.6 div.	+1.8 μ
" 28	+538.8 div.	=	a — 1.07 b	+501.5	+ 2.3 div.	+1.1 μ
" 28	+505.0 div.	=	a — 0.11 b	+501.2	+ 2.0 div.	+1.0 μ

Normal Equations.

$$+5942.1 = 21\,a + 130.51\,b \qquad b = -34.82$$
$$-67160.4 = +130.51\,a + 3800.56\,b \qquad a = +499.2$$

EQUATIONS OF CONDITION BETWEEN METERS S AND $C.S.$

Series II.

1883.	$S - C.S$		$(\tau - 0^{\circ})$	a	Δa	Δa
Feb. 28	+514.3 div.	=	$a - 0.80\,b$	+486.5	+ 1.4 div.	+0.7 μ
" 28	+566.0 div.	=	$a - 2.33\,b$	+485.0	— 0.1 div.	—0.1 μ
" 28	+614.7 div.	=	$a - 3.76\,b$	+484.0	— 1.1 div.	—0.5 μ
" 28	+378.3 div.	=	$a + 3.13\,b$	+487.2	+ 2.1 div.	+1.0 μ
Mar. 1	+286.0 div.	=	$a + 5.65\,b$	+482.4	— 2.7 div.	—1.4 μ
" 3	—185.8 div.	=	$a + 19.27\,b$	+484.0	— 1.1 div.	—0.5 μ
" 4	+832.7 div.	=	$a - 10.03\,b$	+484.0	— 1.1 div.	—0.6 μ
" 4	+806.1 div.	=	$a - 9.09\,b$	+490.0	+ 4.9 div.	+2.4 μ
" 4	+764.6 div.	=	$a - 8.06\,b$	+484.4	— 0.7 div.	—0.3 μ
" 4	+646.0 div.	=	$a - 4.40\,b$	+493.6	+ 8.5 div.	+4.2 μ
" 4	+515.7 div.	=	$a - 0.89\,b$	+484.8	— 0.3 div.	—0.1 μ
" 5	+909.8 div.	=	$a - 12.27\,b$	+483.2	— 1.9 div.	—0.9 μ
April 18	+181.4 div.	=	$a + 8.85\,b$	+489.1	+ 4.0 div.	+2.0 μ
" 19	+249.7 div.	=	$a + 6.70\,b$	+482.7	— 2.4 div.	—1.2 μ
" 19	—307.0 div.	=	$a + 22.88\,b$	+488.5	+ 3.4 div.	+1.7 μ
" 20	—132.4 div.	=	$a + 17.79\,b$	+488.6	+ 3.5 div.	+1.7 μ
" 21	—100.9 div.	=	$a + 16.79\,b$	+482.9	— 2.2 div.	—1.1 μ
" 22	+340.7 div.	=	$a + 4.07\,b$	+482.2	— 2.9 div.	—1.5 μ
" 22	—368.3 div.	=	$a + 24.48\,b$	+482.9	— 2.2 div.	—1.1 μ
" 23	+284.5 div.	=	$a + 5.79\,b$	+485.8	+ 0.7 div.	+0.4 μ
" 23	+282.9 div.	=	$a + 5.88\,b$	+487.3	+ 2.2 div.	+1.1 μ
" 24	+285.3 div.	=	$a + 5.77\,b$	+485.9	+ 0.8 div.	+0.4 μ
" 24	—216.9 div.	=	$a + 20.31\,b$	+489.3	+ 4.2 div.	+2.1 μ
" 25	+407.9 div.	=	$a + 2.23\,b$	+485.4	+ 0.3 div.	+0.1 μ
" 25	—218.1 div.	=	$a + 20.39\,b$	+490.8	+ 5.7 div.	+2.8 μ
" 25	—235.1 div.	=	$a + 20.82\,b$	+488.8	+ 3.7 div.	+1.8 μ
" 26	+425.5 div.	=	$a + 1.54\,b$	+479.0	— 6.1 div.	—3.0 μ
" 26	—396.0 div.	=	$a + 25.57\,b$	+493.1	+ 8.0 div.	+4.0 μ
" 27	+313.1 div.	=	$a + 4.97\,b$	+485.9	+ 0.8 div.	+0.4 μ
" 27	—400.1 div.	=	$a + 25.57\,b$	+489.0	+ 3.9 div.	+2.0 μ
" 28	—163.7 div.	=	$a + 18.71\,b$	+487.8	+ 2.7 div.	+1.4 μ
" 29	+206.9 div.	=	$a + 8.08\,b$	+487.8	+ 2.7 div.	+1.3 μ
" 29	—465.4 div.	=	$a + 27.28\,b$	+483.1	— 2.0 div.	—1.0 μ
" 30	+401.8 div.	=	$a + 1.93\,b$	+468.9	—16.2 div.	—8.1 μ
May 1	+290.4 div.	=	$a + 5.12\,b$	+468.4	—16.7 div.	—8.4 μ
" 1	— 88.8 div.	=	$a + 16.69\,b$	+491.5	+ 6.4 div.	+3.2 μ
" 2	+214.5 div.	=	$a + 7.55\,b$	+477.0	— 8.1 div.	—4.1 μ
" 2	—148.0 div.	=	$a + 18.09\,b$	+481.0	— 4.1 div.	—2.0 μ
" 2	—177.0 div.	=	$a + 18.99\,b$	+483.3	— 1.8 div.	—0.9 μ
" 2	—176.9 div.	=	$a + 19.15\,b$	+488.9	+ 3.8 div.	+1.9 μ

1883.	$S - C.S.$		$(\tau - 0^{\circ})$	a	Δa	Δa
May 3	—161.3 div.	=	a +18.47 b	+480.9	—4.2 div.	—2.1 μ
" 3	—137.3 div.	=	a +17.82 b	+482.3	—2.8 div.	—1.4 μ
" 3	—142.1 div.	=	a +18.21 b	+491.1	+6.0 div.	+3.0 μ
" 4	—154.9 div.	=	a +18.35 b	+483.1	—2.0 div.	—1.0 μ
" 6	— 80.2 div.	=	a +16.50 b	+493.5	+8.4 div.	+4.2 μ
" 6	—123.6 div.	=	a +17.46 b	+483.5	—1.6 div.	—0.8 μ
" 6	— 92.7 div.	=	a +16.71 b	+488.3	+3.2 div.	+1.6 μ
" 7	+205.6 div.	=	a + 7.90 b	+480.2	—4.9 div.	—2.5 μ
" 7	—542.0 div.	=	a +29.80 b	+494.2	+9.1 div.	+4.5 μ

Normal Equations.

$$+5709.7 = 49\,a + 519.64\,b \qquad b = -34.77$$
$$-134583.1 = +519.64\,a + 11121.80\,b \qquad a = +485.1$$

Combining results we have, giving the value of b in the first series a weight of 1, and in the second series a weight of 3,

$$b = -34.78 \text{ div.} = -17.53\,\mu.$$

COMPARISON OF LINE-METER $R_2^{a_2}$ WITH END-METER S IN MELTING ICE WITH ONE-INCH OBJECTIVE.

(1 div. = 504 μ.)

SERIES I.

Date.	Y 61	$R_2^{a_2} - S$	Date.	Y 61	$R_2^{a_2} - S$
1883.	°	div.	1883.	°	div.
Feb. 11	+12.74	+ 14.2	Feb. 13	— 7.57	+718.4
"	+13.14	+ 9.8			
"	+13.10	+ 6.0	Feb. 14	+ 1.80	+390.8
			"	+ 1.86	+398.7
Feb. 12	+ 0.54	+433.2			
"	+ 0.64	+431.2	Feb. 15	+ 5.10	+282.5
"	— 1.13	+495.4	"	+ 4.98	+284.0
"	— 1.40	+497.8	"	+ 4.85	+283.0
"	— 1.30	+489.0	"	+ 4.84	+288.0
"	— 1.58	+498.5			
			Feb. 16	— 2.32	+518.3
			"	— 2.32	+520.1
Feb. 13	+ 1.40	+488.0	"	— 2.37	+518.3
"	+ 1.50	+489.6	"	— 2.40	+517.7
"	+ 1.68	+489.3	"	+ 3.68	+320.1
"	+ 1.57	+491.6	"	+ 3.69	+324.7
"	+ 1.57	+486.9	"	+ 3.72	+320.8
"	— 6.80	+685.0			
"	— 7.10	+681.8	Feb. 18	+28.11	+532.9
"	— 7.26	+702.0	"	+28.14	+534.1
"	— 7.17	+698.4	"	+28.28	+541.9
"	— 7.38	+710.8	"	+29.06	—545.5
"	— 7.68	+711.8	"	+29.08	—552.3

SERIES I. — Continued.

Date.	Y 61	$R_2^{a_2} - S$	Date.	Y 61	$R_2^{a_2} - S$
1883.	°	div.	1883.	°	div.
Feb. 18	+28.56	—555 8	Feb. 26	+21.00	—269.0
"	+28.56	—541.1			
"	+24.06	—403.4	Feb. 27	+ 3.22	+331.5
"	+24.04	—397.0	"	+ 3.20	+343.5
"	+24.05	—395.6	"	+ 3 20	+345.5
			"	+ 2.10	+368.9
Feb. 25	+ 8.44	+169.8	"	+ 2.12	+370.7
"	+ 8.54	+168 1	"	+ 2.00	+379.1
"	+ 8.54	+167.9	"	+ 2.00	+380.7
			"	+ 2.10	+368.9
Feb. 26	+20.36	—259.2	"	+ 2.10	+370.7
"	+20.40	—268.0			
"	+20.40	—268.0	Feb. 28	— 4.47	+605.3
"	+20.34	—249 3	"	— 4.47	+598.8
"	+20.90	—262.7	"	— 4.47	+599 7
"	+20.90	—258.6	"	— 4.47	+595.1
"	+21.00	—263.7	"	— 4.47	+594.9

SERIES II.

Date.	Y 61	$R_2^{a_2} - S$	Date.	Y 61	$R_2^{a_2} - S$
1883.	°	div.	1883.	°	div.
Mar. 1	+ 3.24	+324.3	Mar. 4	+19.88	—250.4
"	+ 3.24	+323.4	"	+19.88	—248.6
"	+ 3.30	+323.0	"	+19.88	—242.3
"	+ 3.30	+323.4	"	+19.88	
"	+ 3.34	+319.6			
"	+ 3.34	+319.7	Mar. 6	+16.46	—123.8
			"	+16.46	—125.2
Mar. 2	+ 4.42	+283.4	"	+16.46	—129.9
"	+ 4.44	+283.5	"	+16.46	—130.5
"	+ 4.44	+279.0	"	+16.54	—135.7
"	+ 4.44	+281.4	"	+16.54	—134.0
"	+ 4.54	+274 0			
"	+ 4.54	+270.6	Mar. 7	+ 8.10	+151.2
"	+ 4.72	+270.4	"	+ 8.20	+148.2
"	+ 4.72	+268.1	"	+ 8.40	+144.8
			"	+ 8.46	+141.8
Mar. 3	+19.18	—212.8	"	+29 92	—587.2
"	+19.18	—215.8	"	+29.92	—583.0
"	+19.18	—213.3	"	+29.92	—592.8
"	+19.18	—213.4	"	+29.92	—593.2
			"	+29.88	—592 6
Mar. 4	—11.40	+810.7	"	+29.88	—590.0
"	—11.35	+813.2	"	+29.98	—596.8
"	—11.35	+796.2	"	+29.98	—598.4
"	—11.45	+800.0	"	+29.98	—589.1
"	+18.00	—166.5	"	+29.98	—595.7
"	+18.00	—168.3	"	+29.98	—594.5
"	+18.00	—177.2			
"	+18.00	—172.0	Mar. 8	+12.90	— 18 4
"	+18.00	—172.7	"	+12.90	— 17.2
"	+18.00	—173.4	"	+12.90	— 10.5
"	+18.00	—174.2	"	+12.90	— 14.0
"	+19.72	—244.1	"	+12.90	— 16 3
"	+19.72	—241.4	"	+12.90	— 15.0

EQUATIONS OF CONDITION BETWEEN METERS $R_2^{s_2}$ AND S.

SERIES I.

1883.	$R_2^{s_2} - S$			$(\tau - 0^\circ)$	a	Δa	Δa
Feb. 11	+ 10.0 div.	=	a	+13.01 b	+460.7	+ 9.9 div.	+4.5 μ
" 12	+432 2 div.	=	a	+ 0.58 b	+452.3	+ 1.5 div.	+0.7 μ
" 12	+495.2 div.	=	a	— 1.31 b	+449.8	— 1.0 div.	—0.5 μ
" 13	+389.1 div.	=	a	+ 1.52 b	+441.8	— 9 0 div.	—4.5 μ
" 13	+701.2 div.	=	a	— 7.10 b	+455.3	+ 4.5 div.	+2.3 μ
" 14	+394.7 div.	=	a	+ 1.81 b	+457.4	+ 6.6 div.	+3.3 μ
" 15	+284.4 div.	=	a	+ 4.89 b	+453.8	+ 3.0 div.	+1.5 μ
" 16	+518.6 div.	=	a	— 2.28 b	+439.6	—11.2 div.	—5.6 μ
" 16	+321.9 div.	=	a	+ 3 65 b	+448.3	— 2.5 div.	—1.3 μ
" 18	—536.3 div.	=	a	+28.33 b	+445.1	— 5.7 div.	—2.8 μ
" 18	—548.7 div.	=	a	+28.97 b	+454.8	+ 4.0 div.	+2.0 μ
" 18	—398.3 div.	=	a	+24.16 b	+438.6	—12.2 div.	—6.1 μ
" 25	+168.6 div.	=	a	+ 8.48 b	+462.3	+11.5 div.	+5.7 μ
" 26	—261.1 div.	=	a	+20.46 b	+447.6	— 3.2 div.	—1.6 μ
" 26	—263.5 div.	=	a	+21.03 b	+465.0	+14.2 div.	+7.1 μ
" 27	+340.2 div.	=	a	+ 3.17 b	+450.0	— 0.8 div.	—0.4 μ
" 27	+373.2 div.	=	a	+ 2.05 b	+444.2	— 6.6 div.	—3.3 μ
" 28	+598.8 div.	=	a	— 4.35 b	+448.1	— 2.7 div.	—1.8 μ

Normal Equations.

$$+3020.2 = 18\,a + 147.07\,b \qquad b = -34.64$$
$$-53584.6 = 147.07\,a + 3461.25\,b \qquad a = +450.8$$

EQUATIONS OF CONDITION BETWEEN METERS $R_2^{s_2}$ AND S.

SERIES II.

1883.	$R_2^{s_2} - S$			$(\tau - 0^\circ)$	a	Δa	Δa
Mar. 1	+322.2 div.	=	a	+ 3.25 b	+432.4	+ 2.0 div.	+1.0 μ
" 2	+276 3 div.	=	a	+ 4.47 b	+427.9	— 2.5 div.	—1.2 μ
" 3	—213.8 div.	=	a	+19.24 b	+438.8	+ 8.4 div.	+4.2 μ
" 4	+805.0 div.	=	a	—10.97 b	+432.9	+ 2.5 div.	+1.2 μ
" 4	—172.0 div.	=	a	+18 05 b	+441.3	+10.9 div.	+5.5 μ
" 4	—242.7 div.	=	a	+19.78 b	+428.2	— 2.2 div.	—1.1 μ
" 4	—247.1 div.	=	a	+19.94 b	+429.3	— 1.1 div.	—0.6 μ
" 6	—129.9 div.	=	a	+16.52 b	+430.5	+ 0.1 div.	+0.1 μ
" 7	+146.6 div.	=	a	+ 8.25 b	+426.5	— 3.9 div.	—1.9 μ
" 7	—589.8 div.	=	a	+30.07 b	+430.2	— 0.2 div.	—0.1 μ
" 7	—594.8 div.	=	a	+30.14 b	+427.4	— 3.0 div.	—1.5 μ
" 8	— 15.2 div.	=	a	+12.92 b	+423.0	— 7.4 div.	—3.7 μ

Normal Equations.

$$-655.3 = 12\,a + 171.66\,b \qquad b = -33.92$$
$$-60293.1 = +171.66\,a + 3956.23\,b \qquad a = +436.4$$

Combining results, giving to the value of b the same weight in each series, we have

$$b = -34.28 \text{ div.} = -17.28\,\mu.$$

COMPARISON OF METERS T^{a_2}, T^{b_1}, *C. S.*, $R_1^{a_2}$, $R_1^{b_2}$, AND $R_2^{a_2}$ WITH UNIVERSAL COMPARATOR.

(Objective = 1 inch.)

(1 div. = 0.440 μ = .0001732 in.)

Date.	Y 61	$T^{a_2}-T^{b_1}$	$T^{a_2}-C.S.$	$T^{a_2}-R^{a_2}$	$T^{a_2}-R^{b_2}$	$T^{a_2}-R_2^{a_2}$
1883.	°	div.	div.	div.	div.	div.
Mar. 22	— 2.21	—13.8	+481.9	+125.8	....	+422.6
" 22	— 1.65	—13.8	+471.1	+126.5	....	+416.6
" 23	— 2.66	—13.1	+488.2	+124.4	....	+431.1
" 23	— 0.62	—14.0	+477.0	+154.1	....	+428.0
" 23	— 0.62	—13.7	+476.4	+151.3	....	+417.5
" 25	— 0.64	—16.4	+477.0	+149.6	....	+422.2
" 25	+ 0.14	—15.2	+477.8	+158.5	....	+424.0
" 25	+ 1.74	—15.5	+470.5	+175.1	....	+408.7
" 25	+ 2.20	—14.6	+468.5	+176.4	....	+413.4
" 25	+ 2.67	—15.8	+465.1	+179.8	....	+402.6
" 25	+ 2.45	—16.0	+462.1	+183.2	....	+407.7
" 26	+ 0.63	—14.0	+469.0	+150.9	....	+413.0
" 26	+ 0.85	—14.0	+469.2	+164.3	....	+409.0
" 26	+ 1.27	—14.4	+475.1	+174.0	....	+415.1
" 28	+ 6.40	—13.1	+463.0	+248.3	....	+404.7
" 29	+ 0.63	—14.0	+459.0	+149.9	....	+403.0
" 29	+ 0.42	—17.3	+467.4	+154.7	....	+410.0
" 29	+ 1.21	—15.1	+473.5	+171.2	....	+415.7
" 29	+ 1.92	—15.2	+473.4	+188.9	....	+414.2
" 30	— 0.48	—15.2	+472.4	+150.6	....	+415.5
" 30	— 0.29	—15.2	+475.2	+147.9	....	+419.0
" 30	+ 1.25	—13.3	+470.4	....	....	+416.1
Apr. 1	+ 0.10	—13.1	+469.6	+155.8	....	+408.7
" 1	+ 0.06	—15.7	+474.7	+162.0	....	+415.1
" 1	+ 0.04	—19.9	+462.6	+155.1	....	+407.9
" 1	+ 0.26	—13.7	+473.0	....	....	....
" 2	— 0.33	—13.4	+470.7	....	....	....
" 2	— 0.26	—16.8	+477.1	....	....	....
" 2	— 0.23	—15.8	+474.1	....	....	....
" 2	— 0.14	—14.8	+473.9	....	....	....
" 2	— 0.14	—13.7	+478.2	+150.3	....	+409.2
" 3	+ 3.98	—15.6	+473.8	+214.6	....	+409.5
" 3	+ 4.39	—13.4	+464.0	+218.4	....	+412.3
" 3	+ 4.63	—15.6	+468.1	+225.2	....	+406.8
" 3	+ 5.02	—14.4	+462.8	+228.9	....	+404.8
" 8	+13.24	—12.1	+424.6	+328.2	....	+379.8
" 8	+12.86	—16.8	+430.1	+332.4	....	+382.2
" 8	+12.23	—13.4	+441.1	+326.5	....	+387.4
" 8	+11.99	—13.1	+439.3	+324.0	....	+391.6
" 9	+10.54	—14.6	+447.7	+310.9	....	+395.6
" 9	+10.26	—14.8	+441.9	+299.8	....	+393.6
" 9	+10.23	—13.0	+444.5	+302.3	....	+393.7
" 10	+ 6.98	—12.6	+445.1	+249.5	....	+392.2
May 14	+14.68	—13.6	+435.9	+349.4	+356.4	+390.1
" 14	+14.68	—13.6	+436.3	+359.0	+346.4	+385.7

COMPARISON OF METERS. — Continued.

Date.	Y 61	$T^{a_2}-T^{b_1}$	$T^{a_2}-C.S.$	$T^{a_2}-R_1^{a_2}$	$T^{a_2}-R_1^{b_2}$	$T^{a_2}-R_2^{a_2}$
1883.	°	div.	div.	div.	div.	div.
May 15	+14.68	—20.4	+427.1	+356.4	+349.4	+386.1
" 15	+14.68	—12.9	+431.4	+369.2	+361.7	+382.0
" 16	+15.02	—14.3	+433.0	+369.6	+363.8	+389.4
" 17	+14.48	—16.4	+426.8	+356.5	+346.9	+383.3
" 17	+15.02	—18.2	+423.4	+369.1	+371.8	+377.9
" 18	+14.46	—15.9	+421.4	+364.3	+357.6	+377.0
" 19	+14.54	—15.0	+424.2	+353.8	+345 8	+381.7
" 20	+14.33	—18.6	+423.2	+352.7	+344.3	+383.2
" 20	+14.46	—13.3	+427.4	+352.4	+346.0	+384.5
" 20	+14.49	—17.0	+423.4	+363.4	+354.9	+384.9
" 21	+14.66	—16.5	+422.5	+357.4	+347.3	+381.0
" 21	+14.66	—16.3	+425.9	+357.9	+348.6	+380.5
" 22	+14.84	—16.6	+426.6	+363.4	+354.5	+384.7
" 23	+14.58	—17.3	+420.3	+354.2	+344.6	+382.8
" 23	+14.58	—16.4	+424.2	+355.3	+347.4	+385.6
" 24	+14.46	—18.4	+421.6	+343.5	+336.6	+374.5
" 24	+14.54	—16.8	+424.4	+348.6	+340.0	+376.8
" 24	+14.48	—16.0	+425.4	+353 3	+344.8	+381.1
" 25	+14.26	—14.6	+430 2	+362.2	+352.5	+389.3
" 25	+14.28	—15 9	+427.7	+349.6	+342 9	+382.0
" 27	+14.28	—18.2	+424.4	+350.6	+342.8	+382.4
" 27	+14.54	—18.6	+425.0	+354.7	+347.0	+383.5
" 28	+14.64	—17.7	+426.9	+356.2	+347.8	+381.0
" 29	+14.76	—18.0	+424.9	+356.4	+349.1	+380.6
" 30	+14.74	—18.3	+428.9	+366.9	+357.6	+388.5
" 30	+14.84	—15.1	+420.4	+354.8	+346.3	+378.7
" 31	+14.74	—16 8	+423.5	+354.7	+345.9	+380.2
" 31	+14.86	—16.6	+418.0	+352.1	+343.4	+376.6
June 1	+14.92	—12.1	+427.7	+370.2	+360.6	+385.6
" 1	+15.08	—16.9	+426.0	+362.0	+352.2	+384.4
" 3	+19.70	—16.5	+404.8	+415.7	+409.1	+362.2
" 3	+19.15	—16 6	+406.2	+417.7	+411.1	+369.2
" 3	+13.69	—15.0	+404.8	+419.3	+412 3	+364.0
" 4	+17.70	—17.9	+411.2	+392.3	+385.4	+375.5
" 4	+17.69	—15.6	+411.9	+397.7	+389.9	+375.8
" 4	+17.48	—14.9	+413.9	+394.9	+387.3	+374.9
" 4	+17.49	—16 5	+416.2	+392.2	+384.0	+376.1
" 5	+17.01	—13.5	+420.4	+393.5	+386.7	+382.0
" 5	+17.13	—16.5	+419 0	+400.2	+394.3	+382.1
" 6	+17.09	—16.1	+420.4	+400.2	+391.5	+377.7
" 7	+17.23	—17.5	+417.4	+400.5	+394.5	+378.5
" 7	+17.23	—17.1	+421.7	+400.4	+393.3	+379.1
" 25	+17.86	—17.0	+413.3	+397.5	+389.8	+368.3
" 26	+17.87	—16.5	+414.4	+402.2	+394.9	+370.1
" 26	+18.04	—15.1	+417.6	+406.6	+399.5	+374.0
" 27	+17.87	—13.9	+414.2	+402.9	+396.2	+371.7
" 27	+17.97	—14.6	+416.4	+408.4	+400 0	+371.6
" 27	+18.16	—16.6	+419.0	+414.1	+404.2	+377.4
" 28	+18.03	—18.0	+416.6	+405.5	+397.6	+373.2

COMPARISON OF YARDS $C.S.$, $R_1^{a_2}$, $R_1^{b_2}$, AND $R_2^{a_3}$ WITH UNIVERSAL COMPARATOR.

(Objective = 1 inch.)

(1 div. = 0.440 μ = .0001732 in.)

Date.	Y 61	$C.S. - R_1^{a_2}$	$C.S. - R_1^{b_2}$	$C.S. - R_2^{a_2}$
1883	°	div.	div.	div.
Mar. 22	− 1.65	−331.2	....	−62.7
" 23	− 2.66	−346.8	....	−58.4
" 23	− 0.62	−321.7	....	−63.6
" 23	− 0.62	−316.6	....	−58.1
" 25	− 0.64	−315.0	....	−61.3
" 25	+ 1.74	−279.9	....	−54.7
" 26	+ 0.63	−303.4	....	−62.8
" 26	+ 0.85	−300.1	....	−62.0
" 26	+ 1.27	−301.1	....	−60.0
" 28	+ 6.40	−208.3	....	−53.7
" 29	+ 0.63	−303.4	....	−62.6
" 29	+ 0.42	−301.5	....	−63.2
" 29	+ 1.21	−283.8	....	−53.8
" 29	+ 1.92	−278.1	....	−57.7
" 30	− 0.48	−315.5	....	−55.8
" 30	− 0.29	−315.2	....	−56.6
" 30	+ 1.25	−284.1	....	−54.7
Apr. 1	+ 0.10	−303.3	....	−52.7
" 1	+ 0.06	−309.6	....	−62.0
" 1	+ 0.04	−309.2	....	−59.9
" 2	− 0.14	−310.4	....	−60.3
" 3	+ 3.98	−252.3	....	−58.0
" 3	+ 4.39	−237.4	....	−57.9
" 3	+ 4.63	−240.3	....	−52.2
" 3	+ 5.02	−233.7	....	−55.2
" 8	+13.24	−105.9	....	−47.9
" 8	+12.86	−115.0	....	−51.9
" 8	+12.23	−118.5	....	−52.1
" 8	+11.99	−122.1	....	−51.2
" 9	+10.54	−153.1	....	−54.8
" 9	+10.26	−144.7	....	−46.9
" 9	+10.23	−150.6	....	−47.0
" 10	+ 6.98	−202.4	....	−52.2
May 14	+14.68	− 89.3	....	−52.4
" 15	+14.48	− 82.7	−83.1	−52.8
" 15	+14.68	− 81.7	−81.1	−50.8
" 16	+15.02	− 80.0	−82.0	−47.8
" 17	+14.48	− 84.6	−84.9	−50.6
" 17	+15.02	− 76.0	−77.9	−50.2
" 18	+14.46	− 78.6	−79.7	−50.5

COMPARISON OF YARDS. — Continued.

Date.	Y 61	$C.S. — R_1^{a_2}$	$C.S. — R_1^{b_2}$	$C.S. — R_2^{a_2}$
1883.	°	div.	div.	div.
May 19	14.54	—84.9	—84.3	—50.3
" 20	14.33	—83.8	—85.8	—50.1
" 20	14.46	—85.3	—87.1	—49.7
" 20	14.49	—86 2	—88.7	—47.5
" 21	14 66	—82.3	—83.4	—48.6
" 21	14.66	—77.4	—79.5	—48.8
" 22	14.84	—85.4	—86.6	—49.2
" 23	14.58	—78.2	—80.8	—53.8
" 23	14.58	—87.5	—89.4	—51.7
" 24	14.46	—86.2	—87.7	—50.7
" 24	14.54	—91.0	—92.4	—45.1
" 24	14.48	—88.4	—88.5	—51.2
" 25	14.26	—95.8	—97.9	—55.6
" 25	14.28	—91.1	—91.9	—52.7
" 27	14.28	—94.0	—95.0	—55.0
" 27	14.54	—84.1	—85.5	—47.2
" 28	14.64	—91.4	—93.6	—50.8
" 29	14.76	—79.7	—82.4	—49.6
" 30	14.74	—86.3	—88.0	—51.7
" 30	14.84	—83.6	—86.6	—51.8
" 31	14.74	—84.9	—85.5	—52.0
" 31	14.86	—84.1	—84.8	—51.2
June 1	14.92	—80.1	—84.5	—50.9
" 1	15.08	—77.2	—78.7	—46.0
" 3	19.70	— 4.4	— 9.4	—48.5
" 3	19.15	— 3.3	— 6.5	—43.7
" 3	18 69	—20.8	—23.0	—47.6
" 4	17.70	—36.8	—39.1	—47.8
" 4	17.69	—41.1	—42.8	—48.6
" 4	17.48	—41.4	—43.2	—49.9
" 4	17.49	—39.5	—40.7	—44.8
" 5	17.01	—43.5	—46.0	—48.4
" 5	17.13	—44.8	—45.3	—47.6
" 6	17.07	—48.5	—40.6	—48.3
" 7	17.23	—46.2	—42.5	—49.1
" 7	17.23	—41.3	—45.5	—48.1
" 25	17.86	—32.0	—32.0	—49.1
" 26	17.87	—34.6	—35.6	—48.6
" 26	18.04	—35.0	—37.0	—48.5
" 27	17.87	—32.2	—34.0	—48.3
" 27	17.97	—29.6	—32.7	—48.7
" 27	18.16	—31.7	—30.7	—48.5
" 28	18.03	—32.1	—30.0	—49.2

EQUATIONS OF CONDITION BETWEEN METRES T^{a_2} AND $C.S.$

1883.	$T^{a}_{2} - C.S.$			$(0° - \tau)$	$b = -3.05$ div. $T^{a_2} - C.S.$ At 0°	Δa	Δa
Mar. 22	+476.5 div.	=	a	— 1.93 b	+470.6 div.	—2.6 div.	—1.1 μ
" 23	+408.5 div.	=	a	— 1.30 b	+476.5 div.	+3.3 div.	+1.5 μ
" 25	+477.4 div.	=	a	— 0.25 b	+476.6 div.	+3.4 div.	+1.5 μ
" 25	+466.6 div.	=	a	+ 2.27 b	+473.5 div.	+0.3 div.	+0.1 μ
" 26	+471.1 div.	=	a	+ 0.92 b	+473.9 div.	+0.7 div.	+0.3 μ
" 28	+463.0 div.	=	a	+ 6.40 b	+472.5 div.	—0.7 div.	—0.3 μ
" 29	+468.4 div.	=	a	+ 1.05 b	+471.6 div.	—1.6 div.	—0.7 μ
" 30	+471.8 div.	=	a	— 0.38 b	+472.6 div.	—0.6 div.	—0.3 μ
" 30	+470.4 div.	=	a	+ 1.25 b	+474.2 div.	+1.0 div.	+0.4 μ
April 1	+468.9 div.	=	a	+ 0.07 b	+469.1 div.	—4.1 div.	—1.8 μ
" 2	+474.2 div.	=	a	— 0.14 b	+473.8 div.	+0.6 div.	+0.3 μ
" 3	+465.0 div.	=	a	+ 4.51 b	+478.8 div.	+5.6 div.	+2.5 μ
" 8	+433.8 div.	=	a	+12.58 b	+472.2 div.	—1.0 div.	—0.4 μ
" 9	+444.7 div.	=	a	+10.34 b	+476.2 div.	+3.0 div.	+1.3 μ
" 10	+445.1 div.	=	a	+ 6.98 b	+466.4 div.	—6.8 div.	—3.0 μ
					At 16°.67		
May 14	+426.1 div.	=	a	+14.68 b	+420.0 div.	+1.0 div.	+0.4 μ
" 15	+429.3 div.	=	a	+14.58 b	+422.9 div.	+3.9 div.	+1.7 μ
" 16	+423.0 div.	=	a	+15.02 b	+418.0 div.	—1.0 div.	—0.4 μ
" 17	+425.1 div.	=	a	+14.75 b	+419.2 div.	+0.2 div.	+0.1 μ
" 18	+421.4 div.	=	a	+14.46 b	+414.7 div.	—4.3 div.	—1.9 μ
" 19	+424.2 div.	=	a	+14.54 b	+417.7 div.	—1.3 div.	—0.6 μ
" 20	+424.5 div.	=	a	+14.43 b	+417.7 div.	—1.3 div.	—0.6 μ
" 21	+424.2 div.	=	a	+14.66 b	+418.1 div.	—0.9 div.	—0.4 μ
" 22	+426.6 div.	=	a	+14.84 b	+421.0 div.	+2.0 div.	+0.9 μ
" 23	+422.3 div.	=	a	+14.58 b	+415.9 div.	—3.1 div.	—1.4 μ
" 24	+423.8 div.	=	a	+14.49 b	+417.2 div.	—1.8 div.	—0.8 μ
" 25	+429.0 div.	=	a	+14.27 b	+421.7 div.	+2.7 div.	+1.2 μ
" 27	+424.7 div.	=	a	+14.41 b	+417.8 div.	—1.2 div.	—0.5 μ
" 28	+426.9 div.	=	a	+14.64 b	+420.7 div.	+1.7 div.	+0.7 μ
" 29	+424.9 div.	=	a	+14.76 b	+419.1 div.	+0.1 div.	+0.0 μ
May 30	+424.7 div.	=	a	+14.79 b	+419.0 div.	+0.0 div.	+0.0 μ
" 31	+420.8 div.	=	a	+14.80 b	+415.1 div.	—3.9 div.	—1.7 μ
June 1	+426.9 div.	=	a	+14.99 b	+421.8 div.	+2.8 div.	+1.2 μ
" 3	+405.8 div.	=	a	+19.18 b	+413.5 div.	—5.5 div.	—2.4 μ
" 4	+413.3 div.	=	a	+17.59 b	+416.1 div.	—2.9 div.	—1.3 μ

1883.	$T^{a_2} - C.S.$			$(0^\circ - \tau)$	$T^{a_2} - C.S.$ At 16°.67	Δa	Δa
June 5	+419.7 div.	=	a	+17.07 b	+420.9 div.	+1.9 div.	+0.8 μ
" 6	+420.4 div.	=	a	+17.07 b	+421.6 div.	+2.6 div.	+1.1 μ
" 7	+419.6 div.	=	a	+17.23 b	+421.4 div.	+2.4 div.	+1.1 μ
" 25	+413.3 div.	=	a	+17.86 b	+416.9 div.	—2.1 div.	—0.9 μ
" 26	+414.4 div.	=	a	+17.87 b	+418.0 div.	—1.0 div.	—0.4 μ
" 26	+417.6 div.	=	a	+18.04 b	+421.8 div.	+2.8 div.	+1.2 μ
" 27	+414.2 div.	=	a	+17.87 b	+417.9 div.	—1.1 div.	—0.5 μ
" 27	+416.4 div.	=	a	+17.97 b	+420.4 div.	+1.4 div.	+0.6 μ
" 27	+419.0 div.	=	a	+18.16 b	+423.5 div.	+4.5 div.	+2.0 μ
" 28	+416.6 div.	=	a	+18.03 b	+420.7 div.	+1.7 div.	+0.7 μ

Normal Equations.

$$+16696.6 = 38\,a + 394.20\,b \qquad b = -3.33$$
$$+167520.3 = 394.20\,a + 5810.50\,b \qquad a = +473.9$$

Since it cannot be assumed that the relative coefficient between T and $C.S.$ for 0° is the same as for 16°.67, it will be advisable to derive the values of a and b from the observations made near these temperatures. From the observations between March 22 and April 10 inclusive, we have

$$+6969.4 = 15\,a + 42.37\,b \qquad b = -3.09$$
$$+18853.5 = 42.37\,a + 389.4\,b \qquad a = +473.3$$

From the observations made between May 14 and June 7, we have

$$+9727.2 = 23\,a + 351.83\,b \qquad b = -3.21$$
$$+148666.8 = 351.83\,a + 5421.10\,b \qquad a = +472.0$$

Whence between 0° and 16°.67,

$$b = -\frac{54.8}{16.67} = -3.29 \text{ div.} = -1.45\,\mu.$$

We have already found,

$$b = 16.22\,\mu - 17.45\,\mu = -1.23\,\mu.$$

Combining, we have,

$$b = \frac{-1.45\,\mu - 1.23\,\mu}{2} = -1.33\,\mu = -3.05 \text{ div.}$$

which is the value which has been used in obtaining the reduced values of $T^{a_2} - C.S.$

EQUATIONS OF CONDITION BETWEEN METERS T^{a_2} AND $R_1^{a_2}$.

1883.	$T^{a_2} - R_1^{a_2}$			$(\tau - 0^\circ)$	$b = +13.80$ div. $T^{a_2} - R_1^{a_2}$ At 0° C.	Δa	Δa
Mar. 22	+126.1 div.	=	a	— 1.93 b	+152.7 div.	—3.0 div.	—1.3 μ
" 23	+143.3 div.	=	a	— 1.30 b	+161.2 div.	+5.5 div.	+2.4 μ
" 25	+154.0 div.	=	a	— 0.25 b	+157.5 div.	+1.8 div.	+0.8 μ
" 25	+178.6 div.	=	a	+ 2.27 b	+147.3 div.	—8.4 div.	—3.7 μ
" 26	+163.1 div.	=	a	+ 0.92 b	+150.4 div.	—5.3 div.	—2.3 μ
" 28	+248.3 div.	=	a	+ 6.40 b	+160.0 div.	+4.3 div.	+1.9 μ
" 29	+166.2 div.	=	a	+ 1.05 b	+151.7 div.	—4.0 div.	—1.8 μ
" 30	+149.2 div.	=	a	— 0.38 b	+154.4 div.	—1.3 div.	—0.6 μ
" 30	+179.7 div.	=	a	+ 1.25 b	+162.4 div.	+6.7 div.	+2.9 μ
April 1	+157.6 div.	=	a	+ 0.07 b	+156.6 div.	+0.9 div.	+0.4 μ
" 2	+150.3 div.	=	a	— 0.14 b	+152.2 div.	—3.5 div.	—1.5 μ
" 3	+221.8 div.	=	a	+ 4.51 b	+159.6 div.	+3.9 div.	+1.7 μ
" 8	+327.7 div.	=	a	+12.58 b	+154.1 div.	—1.6 div.	—0.7 μ
" 9	+304.3 div.	=	a	+10.34 b	+161.6 div.	+5.9 div.	+2.6 μ
" 10	+249.5 div.	=	a	+ 6.98 b	+153.2 div.	—2.5 div.	—1.1 μ
					At 16°.67		
May 14	+354.2 div.	=	a	+14.68 b	+381.6 div.	—5.3 div.	—2.3 μ
" 15	+362.8 div.	=	a	+14.58 b	+391.6 div.	+4.7 div.	+2.1 μ
" 16	+369.6 div.	=	a	+15.02 b	+392.4 div.	+5.5 div.	+2.4 μ
" 17	+362.8 div.	=	a	+14.75 b	+389.3 div.	+2.4 div.	+1.1 μ
" 18	+364.3 div.	=	a	+14.46 b	+394.8 div.	+7.9 div.	+3.5 μ
" 19	+353.8 div.	=	a	+14.54 b	+383.2 div.	—3.7 div.	—1.6 μ
" 20	+356.2 div.	=	a	+14.43 b	+387.1 div.	+0.2 div.	+0.5 μ
" 21	+357.6 div.	=	a	+14.66 b	+385.4 div.	—1.5 div.	—0.7 μ
" 22	+363.4 div.	=	a	+14.84 b	+388.6 div.	+1.7 div.	+0.7 μ
" 23	+354.8 div.	=	a	+14.58 b	+383.6 div.	—3.3 div.	—1.5 μ
" 24	+348.5 div.	=	a	+14.49 b	+378.6 div.	—8.3 div.	—3.7 μ
" 25	+355.9 div.	=	a	+14.27 b	+389.0 div.	+2.1 div.	+0.9 μ
" 27	+352.7 div.	=	a	+14.41 b	+383.9 div.	—3.0 div.	—1.3 μ
" 28	+356.2 div.	=	a	+14.64 b	+384.2 div.	—2.7 div.	—1.2 μ
" 29	+356.4 div.	=	a	+14.76 b	+382.8 div.	—4.1 div.	—1.8 μ
May 30	+360.8 div.	=	a	+14.79 b	+386.7 div.	—0.2 div.	—0.1 μ
" 31	+353.4 div.	=	a	+14.80 b	+379.2 div.	—7.7 div.	—3.4 μ
June 1	+366.1 div.	=	a	+14.99 b	+389.3 div.	+2.4 div.	+1.1 μ
" 3	+417.6 div.	=	a	+19.18 b	+383.0 div.	—3.9 div.	—1.7 μ
" 4	+394.3 div.	=	a	+17.59 b	+381.6 div.	—5.3 div.	—2.3 μ

1883.	$T^{a_2} - R_1^{a_2}$			$(\tau - 0^{\circ})$	$T^{a_2} - R_1^{a_2}$ At 16°.67	Δa	Δa
June 5	+396.8 div.	=	a	+17.07 b	+391.3 div.	+4.4 div.	+1.9 μ
" 6	+400.2 div.	=	a	+17.07 b	+394.7 div.	+7.8 div.	+3.4 μ
" 7	+400.4 div.	=	a	+17.23 b	+392.7 div.	+5.8 div.	+2.6 μ
" 25	+397.5 div.	=	a	+17.86 b	+381.1 div.	—5.8 div.	—2.6 μ
" 26	+404.2 div.	=	a	+17.87 b	+387.6 div.	+0.7 div.	+0.3 μ
" 26	+406.6 div.	=	a	+18.04 b	+387.6 div.	+0.7 div.	+0.3 μ
" 27	+402.9 div.	=	a	+17.87 b	+386.3 div.	—0.6 div.	—0.3 μ
" 27	+408.4 div.	=	a	+17.97 b	+390.4 div.	+3.5 div.	+1.5 μ
" 27	+414.1 div.	=	a	+18.16 b	+394.0 div.	+7.1 div.	+3.1 μ
" 28	+405.5 div.	=	a	+18.03 b	+386.7 div.	—0.2 div.	—0.1 μ

Normal Equations.

$$+11378.5 = +38\,a + 394.2\,b \qquad b = +13.80$$
$$+141584.9 = +394.2\,a + 5810.5\,b \qquad a = +156.2$$

EQUATIONS OF CONDITION BETWEEN METERS T^{a_2} AND $R_2^{a_2}$.

1883.	$T^{a_2} - R_2^{a_2}$			$(\tau - 0^{\circ})$	$b = -2.29$ div. $T^{a_2} - R_2^{a_2}$ At 0° C.	Δa	Δa
Mar. 22	+419.6 div.	=	a	— 1.93 b	+415.2 div.	—0.4 div.	—0.2 μ
" 23	+425.6 div.	=	a	— 1.30 b	+422.6 div.	+7.0 div.	+3.1 μ
" 25	+423.1 div.	=	a	— 0.25 b	+422.4 div.	+6.8 div.	+3.0 μ
" 25	+408.1 div.	=	a	+ 2.27 b	+413.3 div.	—2.3 div.	—1.0 μ
" 26	+412.4 div.	=	a	+ 0.92 b	+414.5 div.	—1.1 div.	—0.5 μ
" 28	+404.7 div.	=	a	+ 6.40 b	+419.4 div.	+3.8 div.	+1.7 μ
" 29	+410.7 div.	=	a	+ 1.05 b	+413.1 div.	—2.5 div.	—1.1 μ
" 30	+417.2 div.	=	a	— 0.38 b	+416.3 div.	+0.7 div.	+0.3 μ
" 30	+416.1 div.	=	a	+ 1.25 b	+419.0 div.	+3.4 div.	+1.5 μ
Apr. 1	+410.6 div.	=	a	+ 0.07 b	+410.7 div.	—4.9 div.	—2.2 μ
" 2	+409.2 div.	=	a	— 0.14 b	+408.9 div.	—6.7 div.	—2.9 μ
" 3	+408.4 div.	=	a	+ 4.51 b	+418.8 div.	+3.2 div.	+1.4 μ
" 8	+385.2 div.	=	a	+12.58 b	+414.0 div.	—1.6 div.	—0.7 μ
" 9	+394.3 div.	=	a	+10.34 b	+418.0 div.	+2.4 div.	+1.1 μ
" 10	+392.2 div.	=	a	+ 6.98 b	+408.3 div.	—7.3 div.	—3.2 μ
					At 16°.67		
May 14	+387.9 div.	=	a	+14.68 b	+383.3 div.	+6.9 div.	+3.0 μ
" 15	+384.1 div.	=	a	+14.58 b	+379.3 div.	+2.9 div.	+1.3 μ
" 16	+389.4 div.	=	a	+15.02 b	+385.6 div.	+9.2 div.	+4.0 μ
" 17	+380.6 div.	=	a	+14.75 b	+376.2 div.	—0.2 div.	—0.1 μ
" 18	+377.0 div.	=	a	+14.46 b	+371.9 div.	—4.5 div.	—2.0 μ

1883.	$T^{a_2} - R_2^{a_2}$			$(\tau - 0^\circ)$	$T^{a_2} - R_2^{a_2}$ At 16°.67	Δa	Δa
May 19	+381.7 div.	=	a	+14.54 b	+376.8 div.	+0.4 div.	+0.2 μ
" 20	+384.2 div.	=	a	+14.43 b	+379.0 div.	+2.6 div.	+1.1 μ
" 21	+380.7 div.	=	a	+14.66 b	+376.1 div.	—0.3 div.	—0.1 μ
" 22	+384.7 div.	=	a	+14.84 b	+380.5 div.	+4.1 div.	+1.8 μ
" 23	+384.2 div.	=	a	+14.58 b	+379.4 div.	+3.0 div.	+1.3 μ
" 24	+377.5 div.	=	a	+14.49 b	+372.5 div.	—3.9 div.	—1.7 μ
" 25	+385.7 div.	=	a	+14.27 b	+380.2 div.	+3.8 div.	+1.7 μ
" 27	+382.9 div.	=	a	+14.41 b	+377.7 div.	+1.3 div.	+0.6 μ
" 28	+381.6 div.	=	a	+14.64 b	+376.9 div.	+0.5 div.	+0.2 μ
" 29	+380.6 div.	=	a	+14.76 b	+376.2 div.	—0.2 div.	—0.1 μ
" 30	+383.1 div.	=	a	+14.79 b	+378.8 div.	+2.4 div.	+1.1 μ
" 31	+378.4 div.	=	a	+14.80 b	+374.1 div.	—2.3 div.	—1.0 μ
June 1	+385.0 div.	=	a	+14.99 b	+381.1 div.	+4.7 div.	+2.1 μ
" 3	+365.1 div.	=	a	+19.18 b	+370.9 div.	—5.5 div.	—2.4 μ
" 4	+375.6 div.	=	a	+17.59 b	+377.7 div.	+1.3 div.	+0.6 μ
June 5	+382.0 div.	=	a	+17.07 b	+382.9 div.	+6.5 div.	+2.9 μ
" 6	+377.7 div.	=	a	+17.07 b	+378.6 div.	+2.2 div.	+1.0 μ
" 7	+378.0 div.	=	a	+17.23 b	+379.3 div.	+2.9 div.	+1.3 μ
" 25	+368.3 div.	=	a	+17.86 b	+371.1 div.	—5.3 div.	—2.3 μ
" 26	+370.1 div.	=	a	+17.87 b	+373.9 div.	—2.5 div.	—1.1 μ
" 26	+374.0 div.	=	a	+18.04 b	+370.9 div.	—5.5 div.	—2.4 μ
" 27	+371.7 div.	=	a	+17.87 b	+369.0 div.	—7.4 div.	—3.3 μ
" 27	+371.7 div.	=	a	+17.97 b	+368.7 div.	—7.7 div.	—3.4 μ
" 27	+377.4 div.	=	a	+18.16 b	+374.0 div.	—2.4 div.	—1.1 μ
" 28	+373.2 div.	=	a	+18.03 b	+370.1 div.	—6.3 div.	—2.8 μ

Normal Equations.

$$+14905.1 = +38\,a + 394.2\,b \qquad b = -2.29 = -1.01\,\mu$$
$$+150711.1 = +394.2\,a + 5810.5\,b \qquad a = +433.6 = +190.8\,\mu$$

EQUATIONS OF CONDITION BETWEEN YARDS *C.S.* AND $R_1^{a_2}$.

1883.	$C.S. - R_1^{a_2}$			$(\tau - 0^\circ)$	$b = +15.57$ div. $C.S. - R_1^{a_2}$ At 0° C.	Δa	Δa
Mar. 22	—331.2 div.	=	a	— 1.93 b	—301.7 div.	—6.8 div.	—3.0 μ
" 23	—328.4 div.	=	a	— 1.30 b	—308.2 div.	—0.3 div.	—0.1 μ
" 25	—315 0 div.	=	a	— 0.64 b	—305.0 div.	—3.5 div.	—7.5 μ
" 25	—279.9 div.	=	a	+ 1.71 b	—307.0 div.	—1.5 div.	—0.7 μ
" 26	—301.5 div.	=	a	+ 0.92 b	—315.8 div.	+7.3 div.	+3.2 μ

1883.	$C.S. - R_1^{a_2}$			$(\tau - 0^{\circ})$	$C.S. - R_1^{a_2}$ At 0° C.	Δa	Δa
Mar. 28	—208.3 div.	=	a	+ 6.40 b	—307.9 div.	—0.6 div.	—0.3 μ
" 29	—291.7 div.	=	a	+ 1.05 b	—308.0 div.	—0.5 div.	—0.2 μ
" 30	—315.2 div.	=	a	— 0.38 b	—309.4 div.	+0.9 div.	+0.4 μ
" 30	—284.1 div.	=	a	+ 1.25 b	—303.6 div.	—4.9 div.	—2.2 μ
Apr. 1	—307.4 div.	=	a	+ 0.07 b	—308.5 div.	0.0 div.	0.0 μ
" 2	—310.4 div.	=	a	— 0.14 b	—308.2 div.	—0.3 div.	—0.1 μ
" 3	—241.1 div.	=	a	+ 4.51 b	—311.3 div.	+2.8 div.	+1.2 μ
" 8	—115.4 div.	=	a	+12.58 b	—311.3 div.	+2.8 div.	+1.2 μ
" 9	—149.5 div.	=	a	+10.34 b	—310.5 div.	+2.0 div.	+0.9 μ
" 10	—202.4 div.	=	a	+ 6.98 b	—311.1 div.	+2.6 div.	+1.1 μ
					At 16°.67		
May 14	— 89.3 div.	=	a	+14.68 b	— 58.3 div.	—5.6 div.	—2.5 μ
" 15	— 82.2 div.	=	a	+14.58 b	— 49.7 div.	+3.0 div.	+1.3 μ
" 16	— 80.0 div.	=	a	+15.02 b	— 54.3 div.	—1.6 div.	—0.7 μ
" 17	— 80.4 div.	=	a	+14.75 b	— 50.5 div.	+2.2 div.	+1.0 μ
" 18	— 78.6 div.	=	a	+14.46 b	— 44.2 div.	+8.5 div.	+3.7 μ
" 19	— 84.9 div.	=	a	+14.54 b	— 51.7 div.	+1.0 div.	+0.4 μ
" 20	— 85.1 div.	=	a	+14.43 b	— 50.2 div.	+2.5 div.	+1.1 μ
" 21	— 79.9 div.	=	a	+14.66 b	— 48.6 div.	+4.1 div.	+1.8 μ
" 22	— 85.4 div.	=	a	+14.84 b	— 56.9 div.	—4.2 div.	—1.8 μ
" 23	— 82.8 div.	=	a	+14.58 b	— 50.3 div.	+2.4 div.	+1.1 μ
" 24	— 88.3 div.	=	a	+14.49 b	— 54.4 div.	—1.7 div.	—0.7 μ
" 25	— 93.4 div.	=	a	+14.27 b	— 56.0 div.	—3.3 div.	—1.5 μ
" 27	— 89.0 div.	=	a	+14.41 b	— 53.8 div.	—1.1 div.	—0.5 μ
" 28	— 91.4 div.	=	a	+14.64 b	— 59.8 div.	—7.1 div.	—3.4 μ
" 29	— 79.7 div.	=	a	+14.76 b	— 50.0 div.	+2.7 div.	+1.2 μ
" 30	— 85.0 div.	=	a	+14.79 b	— 55.7 div.	—3.0 div.	—1.3 μ
" 31	— 84.5 div.	=	a	+14.80 b	— 55.4 div.	—2.7 div.	—1.2 μ
June 1	— 78.7 div.	=	a	+14.99 b	— 52.5 div.	+0.2 div.	+0.1 μ
" 3	— 9.5 div.	=	a	+19.18 b	— 48.6 div.	+4.1 div.	+1.8 μ
" 4	— 39.7div.	=	a	+17.59 b	— 54.0 div.	—1.3 div.	—0.6 μ
June 5	— 44.1 div.	=	a	+17.07 b	— 50.4 div.	+2.3 div.	+1.0 μ
" 6	— 48.5 div.	=	a	+17.07 b	— 54.8 div.	—2.1 div.	—0.9 μ
" 7	— 43.7 div.	=	a	+17.23 b	— 52.4 div.	+0.3 div.	+0.1 μ
" 25	— 32.0 div.	=	a	+17.86 b	— 50.5 div.	+2.2 div.	+1.0 μ
" 26	— 34.6 div.	=	a	+17.87 b	— 53.3 div.	—0.6 div.	—0.3 μ
" 26	— 35.0 div.	=	a	+18.04 b	— 56.3 div.	—3.6 div.	—1.6 μ
" 27	— 32.2 div.	=	a	+17.87 b	— 50.9 div.	+1.8 div.	+0 8 μ
" 27	— 29.6 div.	=	a	+17.97 b	— 49.8 div.	+2.9 div.	+1.3 μ
" 27	— 31.7 div.	=	a	+18.16 b	— 54.9 div.	—2.2 div.	—1.0 μ
" 28	— 32.1 div.	=	a	+18.03 b	— 53.3 div.	—0.6div.	—0.3 μ

Normal Equations.

$$-5685.7 = +38\,a + 394.2\,b \qquad b = +15.53 = +6.83\,\mu$$
$$-32302.0 = +394.2\,a + 5807.1\,b \qquad a = -310.7 = -136.7\,\mu$$

This value of b reduced to the equivalent for one meter becomes $7.48\,\mu$.

But from

$$T - C.\,S. \qquad b = -3.29 \text{ div.}$$
$$T - R_1 \qquad b = +13.80 \text{ div.}$$

Whence from

$$C.\,S. - R_1. \qquad b = +17.09 = +7.52\,\mu$$

We have, therefore, for 1 meter,

$$b = \frac{+7.48\,\mu + 7.52\,\mu}{2} = +7.50\,\mu = +17.04 \text{ div.}$$

For 1 yard,

$$b = \frac{+6.83\,\mu + 6.87\,\mu}{2} = +6.85\,\mu = +15.57 \text{ div.}$$

EQUATIONS OF CONDITION BETWEEN YARDS $C.S.$ AND $R_2^{a_2}$.

1883.	$C.S. - R_2^{a_2}$			$(\tau - 0^\circ)$	$b = +0.69$ div. $C.S. - R_2^{a_2}$ At 0° C.	Δa	Δa
Mar. 22	—62.7 div.	=	a	$-$ 1.93 b	—61.3 div.	+3.0 div.	+1.3 μ
" 23	—60.0 div.	=	a	$-$ 1.30 b	—59.1 div.	+0.8 div.	+0.4 μ
" 25	—61.3 div.	=	a	$-$ 0.64 b	—60.9 div.	+2.6 div.	+1.1 μ
" 25	—54.7 div.	=	a	+ 1.74 b	—55.9 div.	—2.4 div.	—1.1 μ
" 26	—61.6 div.	=	a	+ 0.92 b	—62.2 div.	+3.9 div.	+1.7 μ
" 28	—53.7 div.	=	a	+ 6.40 b	—58.1 div.	—0.2 div.	—0.1 μ
" 29	—59.3 div.	=	a	+ 1.05 b	—54.6 div.	—3.7 div.	—1.6 μ
" 30	—56.2 div.	=	a	$-$ 0.38 b	—55.9 div.	—2.4 div.	—1.1 μ
" 30	—54.7 div.	=	a	+ 1.25 b	—55.6 div.	—2.7 div.	—1.2 μ
Apr. 1	—58.2 div.	=	a	+ 0.07 b	—58.2 div.	—0.1 div.	—0.0 μ
" 2	—60.3 div.	=	a	$-$ 0.14 b	—60.2 div.	+1.9 div.	+0.8 μ
" 3	—55.8 div.	=	a	+ 4.51 b	—59.0 div.	+0.7 div.	+0.3 μ
" 8	—50.8 div.	=	a	+12.58 b	—59.5 div.	+1.2 div.	+0.5 μ
" 9	—49.6 div.	=	a	+10.34 b	—56.7 div.	—1.6 div.	—0.7 μ
" 10	—52.2 div.	=	a	+ 6.98 b	—57.0 div.	—1.3 div.	—0.6 μ
					At $16^\circ.67$		
May 14	—52.4 div.	=	a	+14.68 b	—51.0 div.	—1.9 div.	—0.8 μ
" 15	—51.8 div.	=	a	+14.58 b	—50.4 div.	—1.3 div.	—0.6 μ
" 16	—47.8 div.	=	a	+15.02 b	—46.7 div.	+2.4 div.	+1.1 μ
" 17	—50.4 div.	=	a	+14.75 b	—49.1 div.	0.0 div.	0.0 μ
" 18	—50.5 div.	=	a	+14.46 b	—49.0 div.	+0.1 div.	0.0 μ

1883.	$C.S. - R_2^{a_2}$			$(\tau - 0^\circ)$	$C.S. - R_2^{a_2}$ At 16.°67	Δa	Δa
May 19	—50.3 div.	=	a	$+14.54\,b$	—48.8 div.	+0.3 div.	$+0.1\,\mu$
" 20	—49.1 div.	=	a	$+14.43\,b$	—47.5 div.	+1.6 div.	$+0.7\,\mu$
" 21	—48.7 div.	=	a	$+14\ 66\,b$	—47.3 div.	+1.8 div.	$+0.8\,\mu$
" 22	—49.2 div.	=	a	$+14.84\,b$	—47.9 div.	+1 2 div.	$+0.5\,\mu$
" 23	—52.8 div.	=	a	$+14.58\,b$	—51.3 div.	—2.2 div.	$-1.0\,\mu$
" 24	—49.0 div.	=	a	$+14.49\,b$	—47.5 div.	+1.6 div.	$+0.7\,\mu$
" 25	—54.1 div.	=	a	$+14.27\,b$	—52.4 div.	—3.3 div.	$-1.5\,\mu$
" 27	—51.1 div.	=	a	$+14.41\,b$	—49.5 div.	—0.4 div.	$-0.2\,\mu$
" 28	—50.8 div.	=	a	$+14.64\,b$	—49.4 div.	—0.3 div.	$-0.1\,\mu$
" 29	—49.6 div.	=	a	$+14.76\,b$	—48.3 div.	+0.8 div.	$+0.4\,\mu$
" 30	—51.7 div.	=	a	$+14.79\,b$	—50.5 div.	—1.4 div.	$-0.6\,\mu$
" 31	—51.6 div.	=	a	$+14.80\,b$	—50.4 div.	—1.3 div.	$-0.6\,\mu$
June 1	—48.4 div.	=	a	$+14.99\,b$	—47.3 div.	+1.8 div.	$+0.8\,\mu$
" 3	—46.6 div.	=	a	$+19.18\,b$	—48.3 div.	+0.8 div.	$+0.4\,\mu$
" 4	—47.8 div.	=	a	$+17.59\,b$	—48.4 div.	+0.7 div.	$+0.3\,\mu$
" 5	—48.0 div.	=	a	$+17.07\,b$	—48.3 div.	+0.8 div.	$+0.4\,\mu$
" 6	—48.3 div.	=	a	$+17.07\,b$	—48.6 div.	+0.5 div.	$+0.2\,\mu$
" 7	—48.6 div.	=	a	$+17.23\,b$	—49.0 div.	+0.1 div.	$+0.0\,\mu$
" 25	—49.1 div.	=	a	$+17.86\,b$	—50.0 div.	—0.9 div.	$-0.4\,\mu$
" 26	—48.6 div.	=	a	$+17.87\,b$	—49.4 div.	—0.3 div.	$-0.1\,\mu$
" 26	—48.5 div.	=	a	$+18.04\,b$	—49.5 div.	—0.4 div.	$-0.2\,\mu$
" 27	—48.3 div.	=	a	$+17.87\,b$	—49.1 div.	0.0 div.	$0.0\,\mu$
" 27	—48.7 div.	=	a	$+17.97\,b$	—49.6 div.	—0.5 div.	$-0.2\,\mu$
" 27	—48.5 div.	=	a	$+18.16\,b$	—49.5 div.	—0.4 div.	$-0.2\,\mu$
" 28	—49.2 div.	=	a	$+18.03\,b$	—50.1 div.	—1.0 div.	$-0.4\,\mu$

Normal Equations.

$$-1999.6 = +38\,a + 394.2\,b \qquad b = +0.63 = +0.28\,\mu$$
$$-19663.9 = +394.2\,a + 5807.1\,b \qquad a = -59.2 = -26.0\,\mu$$

From

$$T - R_2 \qquad b = -1.01\,\mu$$
$$T - C.S. \qquad b = -1.34\,\mu$$

Whence from

$$C.S. - R_2 \qquad b = +0.33\,\mu$$

We have, therefore,

For 1 yard, $$b = \frac{+0.28\,\mu + 0.30\,\mu}{2} = +0.29\,\mu = +0.69 \text{ div.}$$

For 1 meter, $$b = \frac{+0.33\,\mu + 0.31\,\mu}{2} = +0.32\,\mu = +0.73 \text{ div.}$$

COMPARISON OF METERS T^{a_2}, T^{b_1}, $R_1^{a_2}$, $R_1^{b_2}$, AND YARDS $R_1^{a_2}$, $R_1^{b_2}$.

Date.	Y61	$T^{a}_2 - T^{b}_1$ Meter.	Δa	Δa	$R_1^{a}{}_2 - R_1^{b_2}$ Meter.	Δa	Δa	$R_1^{a}{}_2 - R_1^{b_2}$ Yard.	Δa	Δa
1883.	°	div.	div.	μ	div.	div.	μ	div.	div.	μ
March 22	− 1.93	−13.8	+1.8	+0.8	....	....	....	....	....	....
" 23	− 1.30	−13.6	+2.0	+0.9	....	....	....	....	....	....
" 25	− 0.25	−15.8	−0.2	−0.1	....	....	....	....	....	....
" 25	+ 2.27	−15.5	+0.1	+0.0	....	....	....	....	....	....
" 26	+ 0.92	−14.1	+1.5	+0.7	....	....	....	....	....	....
" 28	+ 6.40	−13.1	+2.5	+1.1	....	....	....	....	....	....
" 29	+ 1.05	−15.4	+0.2	+0.1	....	....	....	....	....	....
" 30	− 0.38	−15.2	+0.4	+0.2	....	....	....	....	....	....
" 30	+ 1.25	−13.3	+2.3	+1.0	....	....	....	....	....	....
April 1	+ 0.07	−15.6	0.0	+0.0	....	....	....	....	....	....
" 2	− 0.14	−14.9	+0.7	+0.3	....	....	....	....	....	....
" 3	+ 4.51	−14.8	+0.8	+0.4	....	....	....	....	....	....
" 8	+12.58	−13.8	+1.8	+0.8	....	....	....	....	....	....
" 9	+10.34	−14.1	+1.5	+0.7	....	....	....	....	....	....
" 10	+ 6.98	−12.6	+3.0	+1.3	....	....	....	....	....	....
May 14	+14.68	−13.6	+2.0	+0.9	−9.8	−1.8	−0.8	....	....	....
" 15	+14.58	−16.6	−1.0	−0.4	−7.2	+0.8	+0.3	+0.1	+1.4	+0.6
" 16	+15.02	−14.3	+1.3	+0.6	−5.8	+2.2	+1.0	−2.0	−0.7	−0.3
" 17	+14.75	−17.3	−1.7	−0.7	−8.4	−0.4	−0.2	−1.1	+0.2	+0.1
" 18	+14.46	−15.9	−0.3	−0.1	−6.7	+1.3	+0.6	−1.1	+0.2	+0.1
" 19	+14.54	−15.0	+0.6	+0.3	−8.0	0.0	+0.0	+0.6	+1.9	+0.8
" 20	+14.43	−16.3	−0.7	−0.3	−7.8	+0.2	+0.1	−2.1	−0.8	−0.4
" 21	+14.66	−16.4	−0.8	−0.4	−9.7	−1.7	−0.7	−1.6	−0.3	−0.1
" 22	+14.84	−16.6	−1.0	−0.4	−8.9	−0.9	−0.4	−1.2	+0.1	+0.0
" 23	+14.58	−16.8	−1.2	−0.5	−8.7	−0.7	−0.3	−2.2	−0.9	−0.4
" 24	+14.49	−17.1	−1.5	−0.7	−8.0	0.0	+0.0	−1.0	+0.3	+0.1
" 25	+14.27	−15.2	+0.4	+0.3	−8.2	−0.2	−0.1	−1.4	−0.1	+0.0
" 27	+14.41	−18.4	−2.8	−1.2	−7.7	+0.3	+0.1	−1.2	+0.1	+0.1
" 28	+14.64	−17.7	−2.1	−0.9	−8.4	−0.4	−0.2	−2.2	−0.9	−0.4
" 29	+14.76	−18.0	−2.4	−1.1	−7.3	+0.7	+0.3	−2.7	−1.4	−0.6
" 30	+14.79	−16.7	−1.1	−0.5	−8.9	−0.9	−0.4	−2.3	−1.0	−0.4
" 31	+14.80	−16.7	−1.1	−0.5	−8.7	−0.7	−0.3	−0.6	+0.7	+0.3
June 1	+14.99	−16.7	−1.1	−0.5	−9.7	−1.7	−0.7	−2.9	−1.6	−0.7
" 3	+19.18	−16.0	−0.4	−0.2	−6.7	+1.3	+0.6	−3.5	−2.2	−1.0
" 4	+17.59	−16.2	−0.6	−0.3	−7.6	+0.4	+0.2	−1.7	−0.4	−0.2
" 5	+17.07	−15.0	+0.6	+0.3	−6.3	+1.7	−0.7	−1.5	−0.2	−0.1
" 6	+17.07	−16.1	−0.5	−0.2	−8.7	−0.7	−0.3	0.0	+1.3	+0.6
" 7	+17.23	−17.3	−1.7	−0.7	−6.5	+1.5	+0.7	−0.2	+1.1	+0.5
" 25	+17.86	−17.0	−1.4	−0.6	−7.7	+0.3	+0.1	0.0	+1.3	+0.6
" 26	+17.89	−16.5	−0.9	−0.4	−9.5	−1.5	−0.7	−1.0	+0.3	+0.1
" 26	+18.04	−15.1	+0.5	+0.2	−7.1	+0.9	+0.4	−2.0	−0.7	−0.3
" 27	+17.87	−13.9	+1.7	+0.7	−6.7	+1.3	+0.6	−2.0	−0.7	−0.3
" 27	+17.97	−14.6	+1.0	+0.4	−8.4	−0.4	−0.2	−3.1	−1.8	−0.8
" 27	+18.16	−16.6	−1.0	−0.4	−9.9	−1.9	−0.8	+1.0	+2.3	+1.0
" 28	+18.03	−18.0	−2.4	−1.1	−7.9	+0.1	+0.0	+2.1	+3.4	+1.5
Mar. 22 to Mar. 26	....	−14.56	+1.05	+0.46	....	....	....	....	....	....
Mar. 28 to Apr 1	....	−14.52	+1.09	+0.48	....	....	....	....	....	....
Apr. 2 to Apr. 10	....	−14.04	+1.57	+0.51	....	....	....	....	....	....
May 14 to May 18	....	−15.44	+0.17	+0.07	−7.58	+0.45	+0.20	−1.02	+0.24	+0.11
May 19 to May 23	....	−16.22	−0.61	−0.27	−8.62	−0.59	−0.26	−1.30	−0.04	−0.01
May 24 to May 29	....	−17.25	−1.64	−0.72	−7.92	+0.11	+0.05	−1.70	−0.44	−0.19
May 30 to June 4	....	−16.46	−0.85	−0.37	−8.32	−0.29	−0.13	−2.20	−0.94	−0.41
June 5 to June 26	....	−16.38	−0.77	−0.35	−7.74	+0.29	+0.13	−0.54	+0.72	+0.32
June 26 to June 28	....	−15.64	−0.03	−0.01	−8.00	+0.03	+0.01	−0.80	+0.46	+0.20
MEANS........	....	−15.61 = −6.87 μ			−8.03 = −3.53 μ			−1.26 = −0.55 μ		

COMPARISON OF METERS *C. S.*, R_1, AND R_2.

At 0° and at 16.67°.

Date.	T	$C.S. - R_1^{a_2}$	$C.S. - R_1^{a_2}$ At 0° C.	Δa	Δa	$C.S. - R_2^{a_2}$	$C.S. - R_2^{a_2}$ At 0° C.	Δa	Δa
1883.	°	div.	div.	div.	μ	div.	div.	div.	μ
Mar. 22	— 1.93	—350.4	—317.9	—0.3	—0.1	—56.9	—55.4	+ 2.2	+1.0
" 23	— 1.30	—337.2	—315.3	+2.3	+1.0	—54.9	—53.9	+ 3.7	+1.6
" 25	— 0.25	—323.4	—319.2	—1.6	—0.7	—54.3	—54.2	+ 3.4	+1.5
" 25	+ 2.27	—288.0	—326.2	—8.6	—3.8	—58.5	—60.2	— 2.6	—1.1
" 26	+ 0.92	—308.0	—323.5	—5.9	—2.6	—58.7	—59.4	— 1.8	—0.8
" 28	+ 6.40	—204.7	—312.5	+5.1	+2.2	—48.3	—53.1	+ 4.5	+2.0
" 29	+ 1.05	—302.2	—319.9	—2.3	—1.0	—57.7	—58.5	— 0.9	—0.4
" 30	— 0.38	—324.6	—318.2	—0.6	—0.3	—56.7	—56.3	+ 1.3	+0.6
" 30	+ 1.25	—290.7	—311.8	+5.8	+3.8	—54.3	—55.2	+ 2.4	+1.0
Apr. 1	+ 0.07	—311.3	—312.5	+5.1	+2.2	—58.3	—58.4	— 0.8	—0.4
" 2	— 0.14	—323.9	—321.6	—4.0	—1.8	—65.0	—64.9	— 7.3	—3.2
" 3	+ 4.51	—243.2	—319.2	—1.6	—0.7	—56.6	—60.0	— 2.4	—1.1
" 8	+12.58	—106.1	—318.1	—0.3	—0.1	—48.6	—58.2	— 0.6	—0.3
" 9	+10.34	—140.4	—314.6	+3.0	+1.3	—50.4	—58.2	— 0.6	—0.3
" 10	+ 6.98	—195.6	—313.2	+4.4	+1.9	—52.9	—58.1	— 0.5	—0.2
			At 16°.67				At 16°.67		
May 14	+14.68	— 71.9	— 38.4	—6.3	—2.8	—38.2	—36.7	+ 5.9	+2.6
" 15	+14.58	— 66.5	— 31.3	+0.8	+0.4	—45.2	—43.6	— 1.0	—0.4
" 16	+15.02	— 53.4	— 25.6	+6.5	+2.9	—33.6	—32.4	+10.2	+4.5
" 17	+14.75	— 62.3	— 29.9	+2.2	+1.0	—44.5	—43.0	— 0.4	—0.2
" 18	+14.46	— 57.1	— 19.9	+12.2	+5.4	—44.4	—42.8	— 0.2	—0.1
" 19	+14.54	— 70.4	— 34.5	—2.4	—1.1	—42.5	—40.9	+ 1.7	+0.7
" 20	+14.43	— 68.3	— 30.6	+1.5	+0.7	—40.3	—38.7	+ 3.9	+1.7
" 21	+14.66	— 66.6	— 32.7	—0.6	—0.3	—43.5	—42.0	+ 0.6	+0.3
" 22	+14.84	— 63.2	— 32.4	—0.3	—0.1	—41.9	—40.5	+ 2.1	+0.9
" 23	+14.58	— 67.5	— 32.3	—0.2	—0.1	—38.1	—36.5	+ 6.1	+2.7
" 24	+14.49	— 75.3	— 38.6	—6.5	—2.9	—46.3	—44.7	— 2.1	—0.9
" 25	+14.27	— 73.1	— 32.7	—0.6	—0.3	—43.3	—41.5	+ 1.1	+0.5
" 27	+14.41	— 72.0	— 33.9	—1.8	—0.8	—41.8	—40.1	+ 2.5	+1.1
" 28	+14.64	— 70.7	— 36.5	—4.4	—1.9	—45.3	—43.8	— 1.2	—0.5
" 29	+14.76	— 68.5	— 36.3	—4.2	—1.8	—44.3	—42.9	— 0.3	—0.1
" 30	+14.79	— 63.9	— 32.3	—0.2	—0.1	—41.6	—40.2	+ 2.4	+1.1
" 31	+14.80	— 67.4	— 35.9	—3.8	—1.7	—42.4	—41.0	+ 1.6	+0.7
June 1	+14.99	— 60.8	— 32.5	—0.4	—0.2	—41.9	—40.7	+ 1.9	+0.7
" 3	+19.18	+ 11.8	— 30.5	+1.6	+0.7	—40.7	—42.6	+ 0.0	+0.0
" 4	+17.59	— 19.0	— 34.5	—2.4	—1.1	—37.7	—38.4	+ 4.2	+1.8
" 5	+17.07	— 22.9	— 29.6	+2.5	+1.1	—37.7	—38.0	+ 4.6	+2.0
" 6	+17.07	— 20.2	— 26.9	+5.2	+2.3	—42.7	—43.0	— 0.4	—0.2
" 7	+17.23	— 19.2	— 28.7	+3.4	+1.5	—41.6	—42.1	+ 0.5	+0.2
" 25	+17.86	— 15.8	— 35.8	—3.7	—1.6	—45.0	—45.8	— 3.2	—1.4
" 26	+17.89	— 10.2	— 30.4	+1.7	+0.7	—44.3	—44.1	— 1.5	—0.7
" 26	+18.04	— 11.0	— 34.2	—2.1	—0.9	—43.6	—50.9	— 8.3	—3.7
" 27	+17.87	— 11.3	— 31.6	+0.5	+0.2	—42.5	—48.9	— 6.3	—2.8
" 27	+17.97	— 8.0	— 30.0	+2.1	+0.9	—44.7	—51.7	— 9.1	—4.0
" 27	+18.16	— 4.9	— 29.5	+2.6	+1.1	—41.6	—49.5	— 6.9	—3.1
" 28	+18.03	— 11.1	— 34.0	—1.9	—0.8	—43.4	—50.6	— 8.0	—0.4

COMPARISON OF METERS R_1 AND R_2, AND OF YARDS R_1 AND R_2.

At 0° and at 16.67°.

Date.	τ	Meters. $R_1{}^{a}{}_2 - R_2{}^{a}{}_2$	Meters. $R_1{}^{a}{}_2 - R_2{}^{a}{}_2$ At 0° C.	Δa	Δa	Yards. $R_1{}^{a}{}_2 - R_2{}^{a}{}_2$	Yards. $R_1{}^{a}{}_2 - R_2{}^{a}{}_2$ At 0° C.	Δa	Δa
1883.	°			div.	μ			div.	μ
Mar. 22	− 1.93	+293.5	+262.5	+ 2.5	+1.1	+269.1	+240.4	−9.8	−4.3
" 23	− 1.30	+282.3	+261.4	+ 1.4	+0.6	+268.4	+248.9	−1 3	−0.6
" 25	− 0.25	+269.1	+264.9	+ 4.9	+2.2	+253.7	+244.1	−6.1	−2.7
" 25	+ 2 27	+229.5	+266.0	+ 6.0	+2.6	+225.2	+251.1	+0.9	+0.4
" 26	+ 0.92	+249.3	+264.1	+ 4.1	+1.8	+239.9	+253.6	+3.4	+1.5
" 28	+ 6.40	+156.4	+259.4	− 0.4	−0.2	+184 6	+249 8	−0.4	−0.2
" 29	+ 1.05	+244.5	+261 4	+ 1.4	+0.6	+237.8	+253.4	+3.2	+1.5
" 30	− 0.38	+267.9	+261.9	+ 1.9	+0.8	+259.1	+253.5	+3.3	+1.5
" 30	+ 1.25	+236.4	+256.6	− 3.4	−1.5	+229.4	+248.0	−2.2	−1.0
Apr. 1	+ 0.07	+253.0	+254.1	− 5.9	−2.6	+249.2	+250.3	+0.1	+0 0
" 2	− 0.14	+258.9	+256.7	− 3.3	−1.5	+250.1	+247.9	−2 3	−1 0
" 3	+ 4.51	+186.6	+259.2	− 0.8	−0.4	+185 3	+252.3	+2.1	+0.9
" 8	+12.58	+ 57.5	+259.9	− 0.1	+0.0	+ 64 6	+251.8	+1.6	+0.7
" 9	+10.34	+ 90.0	+256.4	− 3.6	−1.6	+ 99.9	+253.8	+3 6	+1.6
" 10	+ 6.98	+142.7	+255.1	− 4.9	−2.2	+150,2	+254.1	+3.9	+1.7
							At 16° 67		
May 14	+14.68	[+ 33.7]	[+ 1.7]	+ . .	[+0.7]	+ 36.9	+ 7.3	+3.4	+1.5
" 15	+14 58	+ 21.3	− 12.3	− 1.3	−0.6	+ 30.4	− 0 7	−4.6	−2.0
" 16	+15.02	+ 19.8	− 6.8	+ 4.2	+1.8	+ 32.2	+ 7.6	+3 7	+1.7
" 17	+14.75	+ 17.8	− 13.1	− 2 1	−0.9	+ 30.0	+ 1.4	−2.5	−1.1
" 18	+14.46	+ 12.7	− 22.9	[−11.9]	[−5.2]	+ 28.1	[− 4.8]	[−8.7]	[−3.8]
" 19	+14.54	+ 27.9	− 6 4	+ 4 6	+2.0	+ 34.6	+ 2.9	−1.0	−0.4
" 20	+14.43	+ 28.0	− 8.1	+ 2.9	+1.3	+ 36.0	+ 2.7	−1.2	−0.5
" 21	+14 66	+ 23.1	− 9.3	+ 1.7	+0.7	+ 31.2	+ 1.3	−2.6	−1.1
" 22	+14.84	+ 21 3	− 8.1	+ 2 9	+1.3	+ 36.2	+ 9.0	+5.1	+2 2
" 23	+14.58	+ 29.4	− 4.2	+ 6.8	+3.0	+ 30.0	− 1.0	−4.9	−2.2
" 24	+14.49	+ 29.0	− 6.1	+ 4.9	+2.2	+ 39.3	+ 6.9	+3.0	+1.3
" 25	+14.27	+ 29.8	− 8.8	+ 2.2	+0 9	+ 39.3	+ 3 6	−0.3	−0 1
" 27	+14.41	+ 30 2	− 6 2	+ 4 8	+2 1	+ 37.9	+ 4 3	+0.4	+0.2
" 28	+14.64	+ 25.4	− 7.3	+ 3.7	+1.6	+ 40.6	+ 10.4	+6.5	+2.9
" 29	+14.76	+ 24.2	− 6.6	+ 4.4	+1.9	+ 30.1	+ 1.7	−2.2	−1.0
" 30	+14.79	+ 22.3	− 7.9	+ 3.1	+1.4	+ 33.3	+ 5.2	+1.3	+0.6
" 31	+14 80	+ 25.0	− 5.1	+ 5.9	+2.6	+ 32.9	+ 5.0	+1.1	+0.5
June 1	+14.99	+ 18.9	− 8.2	+ 2.8	+1 2	+ 30.3	+ 5.2	+1.3	+0.6
" 3	+19.18	− 52.5	− 12.1	− 1.1	−0 5	+ 37.1	+ 0.3	−3.6	−1 6
" 4	+17.59	− 18.7	− 3.9	+ 7.1	+3.1	− 8.1	+ 5 6	+1.7	+0.7
" 5	+17.07	− 14 8	− 8.4	+ 2.6	+1.1	− 3.9	+ 2.1	−1.8	−0.8
" 6	+17.07	− 22.5	− 16.1	− 5.1	−2.2	+ 0.2	+ 6.2	+2.3	+1.0
" 7	+17.23	− 22.4	− 13.4	− 2.4	−1.1	− 4.9	+ 3.4	−0.5	−0.2
" 25	+17.86	− 29.2	− 10 0	+ 1.0	+0.4	− 17.1	+ 0.5	−3.4	−1.5
" 26	+17.89	− 34 1	− 13.7	− 2.7	−1.2	− 14.0	+ 3.9	0.0	+0.0
" 26	+18.04	− 32.6	− 16.7	− 5.7	−2.3	− 13.5	+ 6.8	+2.9	+1.3
" 27	+17 87	− 31 2	− 17.3	− 6 3	−2.8	− 16 1	+ 1.8	−2.1	−0.9
" 27	+17.97	− 36.7	− 21.7	−10.7	−4.7	− 19.1	+ 0.2	−3.7	−1.6
" 27	+18.16	− 36.7	− 20.0	− 9.0	−4.0	− 16.8	+ 5.4	+1 5	+0.7
" 28	+18.03	− 32.3	− 16.6	− 5.6	−5.5	− 17.1	+ 3.2	−0.7	−0.3

COLLECTION OF RESULTS IN GROUPS OF FIVE DATES EACH.

Meters.

At 0° C.

	$T^{a_2}—C.S.$	$T^{a_2}—R_1^{a_2}$	$T^{a_2}—R_2^{a_2}$	$C.S.—R_1^{a_2}$	$C.S.—R_2^{a_2}$	$R_1^{a_2}—R_2^{a_2}$
	div.	div.	div.	div.	div.	div.
	+474.2	+153.8	+417.6	—320.4	—56.6	+263.8
	+472.0	+157.0	+415.7	—315.0	—56.3	+258.7
	+473.5	+156.1	+413.6	—317.4	—59.9	+257.5
Means	+473.2	+155.6	+415.6	—317.6	—57.6	+260.0

At 16°.67 C.

	+419.0	+389.9	+379.8	—29.1	—39.7	—10.6
	+418.1	+385.6	+378.4	—32.5	—39.7	— 7.2
	+419.3	+383.7	+376.7	—35.6	—42.6	— 7.0
	+417.1	+384.0	+376.5	—33.1	—40.6	— 7.5
	+419.8	+389.5	+377.2	—30.3	—42.6	—12.3
	+420.9	+389.0	+370.5	—31.9	—50.4	—18.5
Means	+419.0	+386.9	+376.4	—32.1	—42.6	—10.5

Whence the following relative coefficients are derived:—

For $T—C.S.$ $\quad b = \dfrac{+473.2 — 419.0}{16.67} = +3.25 = +1.43\,\mu$

$T—R_1$ $\quad b = \dfrac{+155.6 — 386.9}{16.67} = —13.87 = —6.10\,\mu$

$T—R_2$ $\quad b = \dfrac{+415.6 — 376.4}{16.67} = +2.35 = +1.03\,\mu$

$C.S.—R_1$ $\quad b = \dfrac{—317.6 + 32.1}{16.67} = —17.12 = —7.53\,\mu$

$C.S.—R_2$ $\quad b = \dfrac{—57.6 + 42.6}{16.67} = —0.90 = —0.40\,\mu$

$R_1—R_2$ $\quad b = \dfrac{+260.0 + 10.5}{16.67} = +16.23 = +7.14\,\mu$

Converting b for $C.S.—R_1$, $C.S.—R_2$, and $R_1—R_2$ into the equivalent values for 1 yard, we have

For $C.S.—R_1$ $\quad b = —6.91\,\mu$

$C.S.—R_2$ $\quad b = —0.36\,\mu$

$R_1—R_2$ $\quad b = +6.54\,\mu$

Yards.

At 0° C.

	$C.S. - R_1^{a_2}$	$C.S. - R_2^{a_2}$	$R_1^{a_2} - R_2^{a_2}$
	div.	div.	div.
	—307.5	—60.0	+247.5
	—307.5	—56.5	+251.0
	—310.5	—58.5	+252.0
Means	—308.5	—58.3	+250.2

At 16°.67 C.

	—51.4	—49.2	+2.2
	—51.5	—48.6	+2.9
	—54.8	—49.4	+5.4
	—53.2	—48.9	+4.3
	—52.3	—49.1	+3.2
	—53.0	—49.6	+3.4
Means	—52.7	—49.1	+3.6

Hence: —

For $C.S.—R_1$ $$b = \frac{-308.5 + 52.7}{16.67} = -15.35 = -6.75\mu$$

$C.S.—R_2$ $$b = \frac{-58.3 + 49.1}{16.67} = -0.55 = -0.24\mu$$

$R_1—R_2$ $$b = \frac{+250.2 - 3.6}{16.67} = +14.80 = +6.51\mu$$

Converting these values into the equivalent values for 1 meter, we have

For	$C.S.—R_1$	$b = -7.38\mu$
	$C.S.—R_2$	$b = -0.27\mu$
	$R_1—R_2$	$b = +7.12\mu$

Combining these results, we have

		For 1 Meter.	For 1 Yard.
For	$T—C.S.$	$b = +1.43\mu$	$b = +1.31\mu$
	$T—R_1$	$b = -6.10\mu$	$b = -5.58\mu$
	$T—R_2$	$b = +1.03\mu$	$b = +0.94\mu$
	$C.S.—R_1$	$b = -7.46\mu$	$b = -6.83\mu$
	$C.S.—R_2$	$b = -0.33\mu$	$b = -0.30\mu$
	$R_1—R_2$	$b = +7.13\mu$	$b = +6.52\mu$

Thus far there is no conclusive evidence that any of these coefficients vary in value with the temperature. For the present, therefore, it will be assumed that they are constant between the limits 0° and 16°.67. In order to compare the relative coefficients for *T*, *C. S.*, and R_2 derived from the absolute coefficients of these bars, it will be better to re-determine absolute coefficients from such observations as are symmetrically disposed about these limits. The following table contains the observations which fall within the limits 0° ± 8° and 16°.67 ± 8°.

The following corrections have been applied to the values under the first series for each bar, in order to reduce them to the system of the second series, viz.:—

For *T*	corr. = —13.8 div.
C. S.	corr. = —14.1 div.
R_2	corr. = —20.4 div.

DETERMINATION OF THE ABSOLUTE COEFFICIENTS OF METERS *T*, *C. S.*, AND R_2, FROM THE NORMAL VALUES FOR 0° AND 16°.67.

For the temperatures (τ — 0°) the reductions to 0° have been made with the values of *b* derived from the normal equations.

The value assumed for (16°.67 — τ) is written at the head of the column.

Bar T. $b = +32.18$ div.

Date.	(τ — 0°)	$S - T^{b_1}$	Means by Groups.	Date.	(16°.67 — τ)	$S - T^{b_1}$	Means by Groups.
1883.	°	div.	div.	1883.	°	div.	div.
Feb. 7	+1.76	+60.7		Feb. 11	+3.20	—476.9	
" 7	+3.83	+57.8		" 19	—7.86	—480.4	
" 7	+7.60	+59.6		" 25	+9.09	—473.2	
" 8	—4.30	+65.1		" 25	+8.36	—469.6	
" 8	+1.46	+53.8	+59.4	" 26	—5.70	—475.6	—475.1
" 9	—4.54	+61.3		Mar. 3	—2.66	—476.8	
" 12	+1.78	+61.4		Apr. 15	—2.74	—472.6	
" 12	—2.96	+59.5		" 17	+4.42	—474.9	
" 12	—0.79	+59.0		" 18	+7.53	—472.9	
" 13	+1.69	+62.2	+60.7	" 19	+8.03	—474.6	—474.4
" 13	+0.12	+60.8		" 20	—1.29	—475.3	
" 13	+3.26	+65.2		" 24	—3.80	—471.6	
" 14	—6.50	+56.5		" 25	—3.37	—478.4	
" 15	+5.82	+57.5		" 26	—4.59	—474.7	
" 16	+0.78	+61.0	+60.2	" 27	+7.65	—475.8	—475.2

Bar T. — Continued.

Date.	$(\tau - 0^\circ)$	$S - T^{b_1}$	Means by Groups.	Date.	$(16\overset{\circ}{.}67 - \tau)$	$S - T^{b_1}$	Means by Groups.
1883.	°	div.	div.	1883.	°	div.	div.
Feb 20	—1.92	+57.0		Apr. 27	+0.30	—475.3	
" 25	+2.96	+63.0		" 28	—1.87	—473.9	
" 27	—6.71	+63.1		" 29	+1.32	—473.1	
" 27	+2.91	+61.5		" 29	+0.66	—475.1	
" 28	—3.47	+63.1	+61.5	" 30	+0.79	—474.4	—474.4
" 28	—0.22	+63.0		" 30	—1.09	—475.0	
" 28	—0.71	+63.5		May 1	—1.01	—472.5	
Mar. 1	+3.64	+62.1		" 1	+0.11	—476.4	
" 4	—0.35	+59.2		" 2	+7.95	—475.1	
" 5	—4.38	+66.7	+62.9	" 4	—1.40	—467.4	—473.3
" 5	—1.18	+65.6		" 4	—1.49	—469.9	
Apr. 22	+4.57	+59.9		" 4	—3.45	—470.8	
" 23	+5.97	+60.9		" 6	+0.20	—472.2	
" 24	+6.02	+59.5		" 7	+8.75	—476.8	
" 25	+5.67	+60.1	+61.2	" 8	+3.80	—473.6	—472.7

Bar C. S. $b = +34.63$ div.

1883.	°	div.		1883	°	div.	
Feb. 12	—6.14	+483.2		Feb. 26	—5.29	—85.7	
" 12	—1.15	+488.8		Mar. 3	—2.60	—95.8	
" 13	+4.69	+490.0		Apr. 18	+7.82	—89.4	
" 13	—7.76	+488.3		" 19	—6.21	—92.0	
" 14	—4.87	+490.2	+488.1	" 20	—1.12	—93.6	—91.3
" 15	+6.14	+481.3		" 21	—0.12	—96.8	
" 16	—1.44	[+473.7]		" 22	—7.81	—97.8	
" 20	—2.96	[+476.7]		" 25	—3.72	—89.3	
" 25	+1.75	+490.4		" 25	—4.15	—91.4	
" 27	+0.67	[+475.4]	+485.8	" 28	—2.04	—93.1	—93.7
" 28	—2.58	+488.7		May 1	—0.02	—88.1	
" 28	—1.07	+487.4		" 2	—1.42	—98.8	
" 28	—0.11	+487.1		" 2	—2.32	—96.7	
" 28	—0.80	+486.5		" 2	—2.48	—91.0	
" 28	—2.33	+485.0	+486.9	" 3	—1.80	—99.0	—94.7
" 28	—3.76	+484.0		" 3	—1.15	—97.5	
" 28	+3.13	+487.2		" 3	—1.54	—88.8	
Mar. 1	+5 65	+482.5		" 4	—1.68	—96.7	
" 4	—8.06	+484.4		" 6	+0.17	—86.1	
" 4	—4.40	+493.6	+486.3	" 6	—0.79	—96.3	
				" 6	—0.04	—91.3	—92.8
" 4	—0.89	+484.8					
" 19	+6.70	+482.7					
Apr. 22	+4.07	+482 2					
" 23	+5.79	+485.8					
" 23	+5.88	+487.3	+484.6				
" 24	+5.77	+485.1					
" 25	+2.23	+485.4					
" 26	+1.54	+479.0					
" 27	+4.97	+485.9					
May 2	+7.55	+477.0	+482.6				

Bar R_2

Date.	$(\tau - 0^\circ)$	$S - T^{b_1}$	Means by Groups.	Date.	$(16^\circ.67 - \tau)$	$S - T^{b_1}$	Means by Groups.
1883.	°	div.	div.	1883.		div.	
Feb. 12	+0.58	+431.9		Feb. 11	+3.66	—135.4	
" 12	—1.31	+429.4		" 25	+8.19	—131.5	
" 13	+1.52	+421.4		" 26	—3.79	[—152.1]	
" 14	+1.81	+437.0		" 26	—4.36	—135.0	
" 15	+4.89	+433.4	+430.6	Mar. 31	—2.57	—126.0	—132.0
" 16	—2.28	+419.2		May 4	—1.38	—124.9	
" 16	+3.65	+427.9		" 4	—3.11	—136.5	
" 27	+3.17	+429.6		" 4	—3.27	—135.4	
" 27	+2.05	+444.2		" 6	+0.15	—135.6	
" 28	—4.35	+427.7	+429.7	" 8	+3.75	—143.3	—135.1
Mar. 2	+4.27	+427.9					
" 3	+3.25	+432.4					
" 7	+8.25	+426.6	+429.0				

Collecting results, we have

For $S - T$ — For 0° C. — For 16°.67.

$a = +61.0$ div. $= +30.7\,\mu$ $\qquad a = -474.2$ div. $= -239.1\,\mu$

Whence $\qquad b = +32.10$ div. $= +16.18\,\mu$

For $S - C.S.$

$a = +488.7$ div. $= +246.3\,\mu$ $\qquad a = -93.1$ div. $= -46.9\,\mu$

Whence $\qquad b = +34.90$ div. $= +17.59\,\mu$

For $S - R_2$

$a = +429.8$ div. $= +216.6\,\mu$ $\qquad a = -133.6$ div. $= -67.3\,\mu$

whence $\qquad b = +33.80$ div. $= +17.04\,\mu$

We have therefore, from these absolute coefficients, the following relative values.

From	$S - T$	$b = +16.18\,\mu$
	$S - C.S.$	$b = +17.59\,\mu$
	$S - R_2$	$b = +17.04\,\mu$
Whence for	$T - C.S.$	$b = +1.41\,\mu$
	$T - R_2$	$b = +0.86\,\mu$
	$C.S. - R_2$	$b = -0.55\,\mu$

Combining these values with the relative values found from the observations with the universal comparator, we have finally the following values for the absolute coefficients: —

Standards.	Coeff. for 1 Meter.	Coeff. for 1 Yard.
T	$+16.18\mu$	$+14.80\mu$
$C.S.$	$\frac{17.59+17.61}{2}=17.60\mu$	16.09μ
R_1	$\frac{10.08+10.14}{2}=10.11\mu$	9.24μ
R_2	$\frac{17.21+17.27+17.04}{3}=17.17\mu$	15.70μ

We are now prepared to place side by side the relations obtained from the comparison of T, $C.S.$, and R_2 with the standard S in melting ice, and the values found by direct comparisons upon the universal comparator. The constant reduction from T^{b_1} to $T^{a_2}=6.9\mu$ is first applied to the observed values $S-T$, $S-C.S.$, and $S-R_2$.

Meters.

	From Comparisons upon the Universal Comparator.				From Comparisons with Bars in Melting Ice.			
	At 0° C.	Wt.	At 16°.67 C.	Wt.	At 0° C.	Wt.	At 16°.67	Wt.
$T^{a_2}-C.S.$	$+208.2\mu$	4	$+184.4\mu$	4	$+208.7\mu$	1	$+185.3\mu$	1
$T^{a_2}-R_1^{a_2}$	$+68.4\mu$	4	$+170.2\mu$	4	....	.	....	.
$T^{a_2}-R_2^{a_2}$	$+182.9\mu$	4	$+165.6\mu$	4	$+179.0\mu$	1	$+164.9\mu$	1
$C.S.-R_1^{a_2}$	-139.7μ	4	-14.1μ	4	....	.	....	.
$C.S.-R_2^{a_2}$	-25.3μ	4	-18.7μ	4	-22.8μ	1	-13.5μ	1
$R_1^{a_2}-R_2^{a_2}$	$+114.4\mu$	4	-4.6μ	4	....	.	....	

Adopted Relations for the Meter.

		At 0° C.		At 16°.67 C.
$T^{a_2}-C.S.$	=	$+208.3\mu$	=	$+184.6\mu$
$T^{a_2}-R_1^{a_2}$		$+68.4\mu$		$+170.2\mu$
$T^{a_2}-R_2^{a_2}$		$+182.1\mu$		$+165.5\mu$
$C.S.-R_1^{a_2}$		-139.7μ		-14.1μ
$C.S.-R_2^{a_2}$		-24.8μ		-17.7μ
$R_1^{a_2}-R_2^{a_2}$		$+114.4\mu$		-4.6μ

Adopted Relations for the Yard.

	At 0° C.	At 16°.67 C.
$C.S.-R_1^{a_2}$	$=-135.7\mu$	$=-23.2\mu$
$C.S.-R_2^{a_2}$	$=-25.6\mu$	$=-21.6\mu$
$R_1^{a_2}-R_2^{a_2}$	$=+110.1\mu$	$=+1.6\mu$

We have now the data for the direct comparison of the independent prototypes T and $C.S.$ of the Metre des Archives, both at 0° and at 16°.67. Representing the Metre des Archives by A, we have:

According to Tresca.	According to Pernet.
$T^{a}_{2}-118.9\mu=A$ at 13°.70 C.	$C.S.+310.0\mu=A$ at 0° C.

With the determined coefficients 16.18 μ and 17.60 μ for T and $C. S.$ respectively, we have

At 0° C.	$T^{a_2} + 102.8\,\mu = A$	
But at 0°	$C. S. + 310.0\,\mu = A$	
Hence	$T^{a_2} - CS =$	$+207.2\,\mu$
From observation	$T^{a_2} - C. S. =$	$+208.3\,\mu$
	Diff. $=$	$1.1\,\mu$

At 16°.67,	$T^{a_2} - 167.0\,\mu = A$	
	$C. S. + 16.6\,\mu = A$	
Hence	$T^{a_2} - C. S. =$	$+183.6\,\mu$
From observation	$T^{a_2} - C. S. =$	$+184.6\,\mu$
	Diff. $=$	$1.0\,\mu$

We have therefore an agreement which is quite as close as one ought to expect.

For the yard we have, from observations at Washington,

$$\text{At } 16°.67,\ R_2 + 1.22\,\mu = Y = \text{Imperial Yard.}$$

From the Report of Mr. Chaney,

	$C. S. + 20.68\,\mu = Y$	
Hence	$C. S. - R_2^{a_2}$	$= -19.5\,\mu$
From observation	$C. S. - R_2^{a_2}$	$= -21.6\,\mu$
	Diff.	$= 2.1\,\mu$

Here again the agreement is extraordinarily close. It may be assumed, therefore, that the meters T and $C. S.$ represent the Metre des Archives within the limits of the ordinary errors of observation. The agreement, also, of the yard $C. S.$ by direct comparison with the Imperial Yard with the relation established through " Bronze 11," indicates that both of these standards represent the Imperial Yard when the determined corrections are applied. In the determination of the length of the meters and yards $R_1^{a_1}$ and $R_2^{a_2}$ it will be assumed that T and $C. S.$, with the determined corrections, represent the original standards with equal weight.

We have, therefore,

At 0° C.

$$T^{a_2} - A = -102.8\,\mu$$

But

$$T^{a_2} - R_1^{a_2} = +\ 68.4\,\mu$$

$$T^{a_2} - R_2^{a_2} = +182.1\,\mu$$

Whence

$$R_1^{a_2} - A = -171.2\,\mu$$

$$R_2^{a_2} - A = -284.9\,\mu$$

Also

$$C.\,S. - A = -310.0\,\mu$$

But

$$C.\,S. - R_1^{a_2} = -139.7\,\mu$$

$$C.\,S. - R_2^{a_2} = -\ 24.8\,\mu$$

Whence

$$R_1^{a_2} - A = -170.3\,\mu$$

$$R_2^{a_2} - A = -285.2\,\mu$$

At 16°.67 C.

$$T^{a_2} - A = +167.0\,\mu$$

But

$$T^{a_2} - R_1^{a_2} = +170.2\,\mu$$

$$T^{a_2} - R_2^{a_2} = +165.5\,\mu$$

Whence

$$R_1^{a_2} - A = \quad -3.2\,\mu$$

$$R_2^{a_2} - A = \quad +1.5\,\mu$$

Also

$$C.\,S. - A = -16.6\,\mu$$

But

$$C.\,S. - R_1^{a_2} = -14.1\,\mu$$

$$C.\,S. - R_2^{a_2} = -17.7\,\mu$$

Whence

$$R_1{}^{a}{}_2 - A = -2.5\,\mu$$

$$R_2^{a_2} - A = +1.1\,\mu$$

We have therefore, finally, by combination,

At 0° C.

$$\boldsymbol{R_1^{a_2} + 170.7\,\mu = A}$$

$$\boldsymbol{R_2^{a_2} + 285.1\,\mu = A}$$

At 16°.67 C.

$$\boldsymbol{R_1^{a_2} + 2.8\,\mu = A}$$

$$\boldsymbol{R_2^{a_2} - 1.3\,\mu = A}$$

It will be observed that the relation for $R_2^{a_2}$ for 16°.67 is nearly identical with that determined in 1880 and 1881.

For the yard we have the following final results.

According to

$\frac{\text{Rogers} + \text{Smith}}{2}$	$R_2^{a_2} - Y$	$= -\ 1.2\,\mu$
Chaney	$C.\,S. - Y$	$= -20.7\,\mu$
From observations	$C.\,S. - \dot{R}_2^{a_2}$	$= -21.6$
Whence	$R_2^{a_2} - Y$	$= +\ 0.9$

And finally,

At 16°.67 C.

$$R_2^{a_2} + 0.2\,\mu \qquad = Y$$

or,

$$R_2^{a_2} + .000008 \text{ in.} = Y$$

For $R_1^{a_2}$ we have, since

$$R_1^{a_2} - R_2^{a_2} \qquad = +1.6\,\mu$$

$$R_1^{a_2} - 1.4\,\mu \qquad = Y$$

or,

$$R_1^{a_2} - .000056 \text{ in.} = Y$$

COMPARISON OF METERS $R_1^{a_2}$, $G_1^{a_2}$, AND $G_2^{a_2}$, WITH UNIVERSAL COMPARATOR.

(Objective = 1 inch. 1 div. = .440 μ.)

Date. 1883.	*Equations of Condition.* $R_1^{a_2} - G_1^{a_2}$		$R_1^{a_2} - G_2^{a_2}$			$(\tau - 0)^\circ$	At 0° C. $b = +4.5$ div. $R_1^{a_2} - G_1^{a_2}$	$R_1^{a_2} - G_2^{a_2}$
Jan. 9	—15.7 div.	....	— 4.6 div.	=	a	+8.506	(—53.9) div.	(—42.8) div.
" 9	—14.6 div.	....	—15.1 div.	=	a	+6.046	(—41.9) div.	(—42.4) div.
" 10	—48.9 div.	....	—46.9 div.	=	a	—4.046	—30.7 div.	—28.7 div.
" 11	—35.2 div.	....	—29.0 div.	=	a	—1.386	—29.0 div.	—22.8 div.
" 12	—66.0 div.	....	—62.9 div.	=	a	—7.666	—31.5 div.	—28.4 div.
" 12	—51.6 div.	....	—51.2 div.	=	a	—7.706	(—16.0) div.	(—15.6) div.
" 12	—53.0 div.	....	—58.4 div.	=	a	—7.566	(—19.0) div.	—24.4 div.
" 12	—49.9 div.	....	—48.2 div.	=	a	—4.536	—29.5 div.	—27.8 div.
" 14	+20.4 div.	....	+17.6 div.	=	a	+11.786	—32.6 div.	—35.4 div.
" 19	+32.7 div.	....	+33.2 div.	=	a	+13.826	—29.5 div.	—29.0 div.
" 21	+18.1 div.	....	+21.4 div.	=	a	+11.856	—35.2 div.	—31.9 div.
							At 16°.67 C.	
Jan. 14	+33.4 div.	....	+35.5 div.	=	a	+15.776	+37.4 div.	+39.5 div.
" 15	+31.4 div.	....	+26.5 div.	=	a	+14.446	+41.4 div.	+36.5 div.
" 17	+48.7 div.	....	+58.9 div.	=	a	+21.116	(+28.7) div.	+38.9 div.
" 17	+57.6 div.	....	+65.7 div.	=	a	+20.926	+38.5 div.	+46.6 div.
" 17	+56.5 div.	....	+66.4 div.	=	a	+20.586	+38.9 div.	+48.8 div.
" 18	+38.3 div.	....	+43.9 div.	=	a	+14.86	+46.4 div.	+52.0 div.
" 18	+51.0 div.	....	+52.3 div.	=	a	+18.92	+40.9 div.	+42.2 div.
July 6	+51.0 div.	....	+54.0 div.	=	a	+18.48	+42.9 div.	+45.9 div.
" 8	+66.1 div.	....	+72.9 div.	=	a	+21.72	+43.4 div.	+50.2 div.
" 8	+62.6 div.	....	+70.5 div.	=	a	+20.78	+44.1 div.	+52.0 div.
" 9	+56.5 div.	....	+66.1 div.	=	a	+20.25	+40.4 div.	+50.0 div.
" 9	+56.5 div.	....	+63.9 div.	=	a	+19.95	+41.7 div.	+49.1 div.

Date. 1883.	$R_1^{a_2} - G_1^{a_2}$		$R_1^{a_2} - G_2^{a_2}$		$(\tau - 0)°$	At 16°.67 C. $R_1^{a_2} - G_1^{a_2}$	$R_1^{a_2} - G_2^{a_2}$
July 10	+53.6 div.	....	+55.9 div.	=	a +19.53	+40.7 div.	+43.0 div.
" 10	+55.1 div.	....	+69.4 div.	=	a +19.49	+42.4 div.	+56.7 div.
" 11	+58.3 div.	....	+63.1 div.	=	a +19.11	+47.3 div.	+52.1 div.
" 11	+54.6 div.	....	+56.1 div.	=	a +19.13	+43.5 div.	+45.0 div.
" 12	+54.0 div.	....	+61.2 div.	=	a +19.20	+42.6 div.	+49.8 div.
" 12	+59.5 div.	....	+62.8 div.	=	a +19.07	+48.7 div.	+52.0 div.
" 13	+53.7 div.	....	+63.5 div.	=	a +19.08	+42.9 div.	+52.7 div.
" 13	+53.4 div.	....	+61.8 div.	=	a +19.08	+42.6 div.	+51.0 div.

Normal Equations.

For $R_1^{a_2} - G_2^{a_2}$

$$+788.1 = 31\,a + 400.6\,b$$
$$+22821.6 = 400.6\,a + 8141.3\,b$$
$$b = -4.27$$
$$a = -29.8$$

For $R_1^{a_2} - G_2^{a_2}$

$$+926.3 = 31\,a + 400.6\,b$$
$$+25256.0 = 400.6\,a + 8141.3\,b$$
$$b = -4.48$$
$$a = -28.0$$

COMPARISON OF YARDS $R_1^{a_2}$, $G_1^{a_2}$, AND $G_2^{a_2}$, WITH UNIVERSAL COMPARATOR.

Equations of Condition.

Date. 1883.	$R_1^{a_2} - G_1^{a_2}$	$R_1^{a_2} - G_2^{a_2}$		$(\tau - 0)°$	At 0° C. $b = +4.1$ $R_1^{a_2} - G_1^{a_2}$	$R_1^{a_2} - G_2^{a_2}$
Jan. 9	−13.2 div.	−10.0	=	$a + 8.50\,b$	(−48.0) div.	(−44.8) div.
" 9	−19.9 div.	− 8.7	=	$a + 6.04\,b$	(−44.7) div.	−33.5 div.
" 10	−42.8 div.	−31.8	=	$a - 4.04\,b$	−26.2 div.	(−15.2) div.
" 11	−26.2 div.	−30.8	=	$a - 1.38\,b$	−20.5 div.	−25.1 div.
" 12	−64.9 div.	−66.7	=	$a - 7.66\,b$	−33.8 div.	−35.3 div.
" 12	−59.7 div.	−61.5	=	$a - 7.70\,b$	−28.1 div.	−29.9 div.
" 12	−61.4 div.	−65.3	=	$a - 7.56\,b$	−30.4 div.	−34.3 div.
" 12	−43.9 div.	−37.3	=	$a - 4.53\,b$	−25.3 div.	−18.7 div.
" 14	+13.2 div.	+ 8.0	=	$a + 11.78\,b$	−35.1 div.	−40.3 div.
" 19	+27.2 div.	+29.3	=	$a + 13.82\,b$	−29.5 div.	−27.4 div.
" 21	+20.0 div.	+23.6	=	$a + 11.85\,b$	−28.6 div.	−25.0 div.
					At 16°.67 C.	
" 14	+29.6 div.	+28.8	=	$a + 15.77\,b$	+33.3 div.	+32.5 div.
" 15	+34.1 div.	+32.5	=	$a + 14.44\,b$	+43.2 div.	+41.6 div.
" 17	+46.4 div.	+59.6	=	$a + 21.11\,b$	(+28.2) div.	+41.4 div.
" 17	+56.2 div.	+57.4	=	$a + 20.92\,b$	+38.8 div.	+40.0 div.
" 17	+53.0 div.	+55.3	=	$a + 20.58\,b$	+37.0 div.	+39.3 div.
" 18	+38.0 div.	+32.9	=	$a + 14.86\,b$	+45.4 div.	+40.3 div.
" 18	+47.2 div.	+52.4	=	$a + 18.92\,b$	+38.0 div.	+43.2 div.
July 6	+48.0 div.	+49.0	=	$a + 18.48\,b$	+40.6 div.	+41.6 div.
" 8	+65.4 div.	+64.1	=	$a + 21.72\,b$	+44.7 div.	+43.4 div.
" 8	+56.0 div.	+60.9	=	$a + 20.78\,b$	+39.1 div.	+44.0 div.

Date. 1883.	$R_1^{a_2} - G_1^{a_2}$	$R_1^{a_2} - G_2^{a_2}$		$(\tau - 0)^\circ$	At 16°.67 C. $R_1^{a_2} - G_1^{a_2}$	$R_1^{a_2} - G_2^{a_2}$
July 9	+56.3 div.	+56.2	=	a +20.25 b	+41.6 div.	+41.5 div.
" 9	+55.7 div.	+53.4	=	a +19.95 b	+42.3 div.	+39.9 div.
" 10	+61.0 div.	+60.8	=	a +19.53 b	+49.3 div.	+49.1 div.
" 10	+51.3 div.	+51.3	=	a +19.41 b	+39.7 div.	+39.7 div.
" 11	+54.7 div.	+56.1	=	a +19.11 b	+44.7 div.	+46.1 div.
" 11	+58.7 div.	+52.0	=	a +19.13 b	+48.6 div.	+41.9 div.
" 12	+50.2 div.	+51.8	=	a +19.20 b	+39.8 div.	+41.4 div.
" 12	+53.5 div.	+54.3	=	a +19.07 b	+43.7 div.	+44.5 div.
" 13	+51.5 div.	+52.2	=	a +19.08 b	+41.6 div.	+42.3 div.
" 13	+54.1 div.	+50.8	=	a +19.08 b	+44.2 div.	+40.9 div.

Normal Equations.

For $R_1^{a_2} - G_1^{a_2}$	For $R_1^{a_2} - G_2^{a_2}$
$+739.3 = 31\,a + 400.6\,b$	$+780.6 = 31\,a + 400.6\,b$
$+22148.0 = 400.6\,a + 8141.3\,b$	$+22476.3 = 400.6\,a + 8141.3\,b$
$b = +4.25$	$b = +4.18$
$a = -31.1$	$a = -28.8$

The two values of b for the yard when reduced to the equivalent for one meter become 4.66 div. and 4.57 div. respectively. The mean value of b, therefore, derived from these equations, is 4.49 div. for one meter, or 4.10 div. for one yard. Substituting these values in the equations, we find a few values of a which differ from the mean value more than four times the probable error of a single value. In the formation of the mean values for 0° and for 16°.67 these quantities have been rejected.

For the meter, we have

	At 0°	At 16°.67	
$R_1^{a^2} - G_1^{a^2} =$	-31.1 div.	$+42.3$ div.	$b = +4.40$ div.
$R_1^{a^2} - G_2^{a^2} =$	-28.6 div.	$+47.7$ div.	$b = +4.58$ div.
		Mean . .	$+4.49$ div.

For the yard, we have

$R_1^{a^2} - G_1^{a^2} =$	-28.6 div.	$+41.9$ div.	$b = +4.23$ div.
$R_1^{a^2} - G_2^{a^2} =$	-29.9 div.	$+41.7$ div.	$b = +4.30$ div.
		Mean . .	$+4.26$ div.

We have, therefore, finally,

$$\text{For 1 meter,} \quad b = \frac{4.49 \text{ div.} + 4.65 \text{ div.}}{2} = +4.57 \text{ div.} = 2.01\,\mu.$$

$$\text{For 1 yard,} \quad b = \frac{4.11 \text{ div.} + 4.26 \text{ div.}}{2} = 4.18 \text{ div.} = 1.84\,\mu.$$

Since the absolute coefficient for R_1 is 10.11 μ, and 9.24 μ for the meter and yard, the absolute coefficient for the glass bars becomes 8.10 μ and 7.40 μ respectively for the meter and the yard.

Since for 16°.67,

For the Meter,		For the Yard,
$R_1^{a_2} + 2.8\,\mu = A$	and	$R_1^{a_2} - 1.4\,\mu = Y$
$R_1^{a_2} - 18.6\,\mu = G_1^{a_2}$		$R_1^{a_2} - 18.4\,\mu = G_1^{a_2}$
$R_1^{a_2} - 21.0\,\mu = G_2^{a_2}$		$R_1^{a_2} - 18.3\,\mu = G_2^{a_3}$

we have, finally,

$$G_1^{a_2} + 21.4\,\mu = A \qquad G_1^{a_2} + 17.0\,\mu = Y$$
$$G_2^{a_2} + 23.8\,\mu = A \qquad G_2^{a_2} + 16.9\,\mu = Y$$

The values which were actually used in the subsequent transfers, derived from a few of the earlier observations, were 21 μ for the meter and 15 μ for the yard.

It will be seen from the following observations that the second transfer was not quite successful, although the magnitude of the residuals is very much reduced. This part of my labor will not be considered as completed until a third transfer has been made, and until advantage can be taken of a lower temperature during the coming winter in order to obtain a more accurate value of the coefficient of expansion of the glass bar.

In the following tables, the values of $R_1^{a_2} - G_2^{b_2}$ and of $R_1^{a_2} - G_2^{d_2}$ have been reduced to $R_1^{a_2} - G_2^{b\frac{123}{3}}$ and $R_2^{a_2} - G_2^{d\frac{123}{3}}$ by applying —0.9 div. and —1.7 div. for the meter, and —0.8 div. and +0.5 div. for the yard, respectively.

COMPARISON OF METERS $R_1^{a_2}$, $G_1^{b_1}$, G_2^{b123}, AND G_2^{d123} WITH ONE-INCH OBJECTIVE.

(1 div. = .440 μ.)

Date.	Y 61	$R_1^{a_2} - G_1^{b_1}$	$R_1^{a_2} - G_2^{b_1}$	$R_1^{a_2} - G_2^{b_2}$	$R_1^{a_2} - G_2^{b_3}$	$R_1^{a_2} - G_2^{d_1}$	$R_1^{a_2} - G_2^{d_2}$	$R_1^{a_2} - G_2^{d_3}$	$b = +4.57$ div. At 16°.67 $R_1^{a_2} - G_1^{b_1}$	$R_1^{a_2} - G_2^{b\frac{123}{3}}$	$R_1^{a_2} - G_2^{d\frac{123}{3}}$
1883.	°	div.		div.			div.		div.	div.	div.
July 8	21.72	+10.0	....	+19.5	....	....	+27.6	....	—13.1	—4.5	+ 2.8
" 8	20.78	+ 3.5	....	+15.9	....	....	+23.6	....	—15.3	—3.8	+ 3.1
" 9	20.25	+ 0.8	....	+11.0	....	....	+22.0	....	—15.6	—6 3	+ 3.9
" 9	19.95	+ 1.2	....	+ 9.6	....	....	+16.4	....	—13.8	—6.3	— 0.3
" 10	19.53	— 1.3	....	+ 8.8	....	....	+10.5	....	—14.4	—5.2	— 4.4
" 10	19.49	— 0.5	+18.6	+13.1	+12.9	+18.5	+22.3	+20.4	—13.4	+1.0	+ 7.5
" 11	19.11	— 2.7	+ 8.1	+11.6	+ 9.4	+11.3	+13.3	+10 3	—13.9	—1.4	+ 0.5
" 11	19.13	— 4.9	+ 9.3	+10.8	+11.2	+18.3	+19.2	+12.7	—16.1	—1.1	+ 4.9
" 12	19.20	— 3 9	+ 8 2	+10.2	+ 7.9	+14.3	+16.0	+12.5	—15.5	—2.8	+ 2.7
" 12	19.07	— 0.3	+ 6.8	+ 8.4	+ 4 2	+12.2	+11.0	+11.9	—11.4	—4.6	+ 0.3
" 13	19.08	— 7.4	+13.2	+13.7	+14.2	+18.6	+21.3	+18.5	—18.4	+2.7	+ 8.5
" 13	19.08	— 6.8	+14.7	+15.4	+15.8	+19.9	+23.1	+19.9	—17.8	+4.3	+10.0
								Means	—14.9	—2.4	+ 3.3
									—6.6μ	—1.1μ	+ 1.5μ

Since $R_1^{a_2} + 2.8\,\mu = A$, we have

$$G_1^{b_1} \quad - 3.8\,\mu = A$$

$$G_2^{b_{\frac{123}{3}}} + 1.7\,\mu = A$$

$$G_2^{d_{\frac{123}{3}}} + 4.1\,\mu = A$$

COMPARISON OF YARDS $R_1^{a_2}$, $G_1^{b_1}$, $G_2^{b_{123}}$, AND $G_2^{d_{123}}$, WITH ONE-INCH OBJECTIVE.

(1 div. = .440 μ.)

									$b = 4.18$ div. At 16°.67		
Date.	Y 61	$R_1^{a_2} - G_1^{b_1}$	$R_1^{a_2} - G_2^{b_1}$	$R_1^{a_2} - G_2^{b_2}$	$R_1^{a_2} - G_2^{b_3}$	$R_1^{a_2} - G_2^{d_1}$	$R_1^{a_2} - G_2^{d_2}$	$R_1^{a_2} - G_2^{d_3}$	$R_1^{a_2} - G_1^{b_1}$	$R_1^{a_2} - G_1^{b\frac{123}{3}}$	$R_1^{a_2} - G_2^{d\frac{123}{3}}$
1883.	°	div.		div.			div.		div.	div.	div.
July 8	21.72	+8.1	....	+32.9	....	....	+40 2	....	−13.0	+10.9	+19.6
" 8	20.78	+0.0	....	+37.2	....	....	+37.9	....	−17.2	+19.1	+21.2
" 9	20.25	−1.6	....	+36.1	....	....	+35.5	....	−16.6	+20.2	+21.0
" 9	19.95	+0.1	....	+23.5	....	....	+23.0	....	−13.8	+10.7	+ 9.7
" 10	19.53	−0.3	....	+15.5	....	....	+23.2	....	−12 5	+ 5.0	+11.0
" 10	19.49	+1.0	+29.1	+27.0	+25.9	+28.5	+27.2	+31.9	−10.8	+15.5	+16.1
" 11	19 11	+0.2	+29.4	+30.3	+26.2	+33.1	+30.3	+32.2	−10.0	+18.4	+22.0
" 11	19.13	+0.5	+31.4	+33.9	+28.4	+35.6	+36 8	+34.1	− 9.8	+20.9	+25.2
" 12	19.20	−5.4	+26.1	+29.1	+22.4	+25.6	+31.3	+35.3	−16.0	+15.5	+19.0
" 12	19.07	−0 3	+24 8	+25.2	+23.7	+24.8	+28.3	+29.5	−10.3	+14.8	+18.6
" 13	19.08	−4.2	+27 0	+26.5	+26.4	+31 7	+28.3	+33.7	−14.2	+16.7	+18.2
" 13	19.08	−2.9	+26.5	+29.9	+28.1	+34.0	+30.9	+35.5	−12.9	+21.2	+23.5
								Means	−13.1	+15.7	+18.8
									− 5.8μ	+ 6.9μ	+ 8.3μ

Since $R_1^{a_2} - 1.4\,\mu = Y$, we have,

$$G_1^{b_1} \quad - 7.2\,\mu = Y$$

$$G_2^{b_{\frac{123}{3}}} + 5.5\,\mu = Y$$

$$G_2^{d_{\frac{123}{3}}} + 6.9\,\mu = Y$$

COMPARISON OF THE WHITWORTH YARD W WITH $R_2^{s_2}$, WITH COMPARATOR NO. 1.

(1 div. = .0000197 in.)

Date. 1881.	Time. h. m.	Fahr. °	$W - R_2^{s_2}$
April 27	5 15 A.M.	57.9	+ 51.7 div.
" 27	6 10 "	58.0	+ 52.1 div.
" 28	5 15 "	61.7	+ 20.8 div.
" 28	3 40 P.M.	67.4	— 25.9 div.
" 28	6 50 "	66.5	— 17.1 div.
" 29	5 30 A.M.	63.9	+ 7.5 div.
" 29	5 35 "	63.9	+ 8.2 div.
" 29	5 44 "	64.7	+ 11.6 div.
" 29	6 0 "	64.0	+ 8.9 div.
" 29	6 10 "	63.9	+ 7.3 div.
" 29	6 22 "	63.9	+ 15.4 div.
" 29	6 45 "	63.7	+ 5.9 div.
" 29	8 10 "	63.4	+ 7.9 div.
" 29	8 15 "	63.4	+ 10.1 div.
" 29	8 30 "	63.4	+ 10.1 div.
" 29	8 50 "	63.8	+ 10.2 div.
May 1	5 30 A.M.	81.4	—127.4 div.
" 1	8 30 "	81.6	—131.7 div.
" 1	8 40 "	81.7	—132.3 div.
" 1	9 15 "	81.8	—130.8 div.
" 1	10 20 "	81.7	—128.7 div.
" 1	11 0 "	81.7	—128.0 div.
" 1	11 45 "	81.7	—129.8 div.
" 1	11 55 "	81.7	—130.0 div.
" 1	12 45 P.M.	81.7	—128.0 div.
" 1	2 0 "	81.6	—128.4 div.
" 2	5 40 A.M.	56.1	+ 65.9 div.
" 2	6 0 "	56.1	+ 66.4 div.
" 2	6 10 "	56.1	+ 63.7 div.
" 2	9 45 "	54.8	+ 75.4 div.
" 3	5 50 "	61.0	+ 25.6 div.

EQUATIONS OF CONDITION BETWEEN W AND $R_2^{a_2}$.

$W - R_2^{a_2}$			$(\tau - 62°)$	At 62° $W - R_2^{a_2}$
+ 16.3 div.	=	a	— 0.3 b	+18.6 div.
+ 9.4 div.	=	a	— 1.8 b	+23.1 div.
—129.5 div.	=	a	—19.7 b	+20.0 div.
+ 59.4 div.	=	a	+ 5.2 b	+19.9 div.
			Mean	+20.4 div.

Normal Equations.

$$-44.4 = 4a - 16.6b \qquad b = +7.59$$
$$+2838.2 = -16.6a + 418.4b \qquad a = +20.4$$

We have, therefore, for 62° Fahr.,

$$W - R_2^{a2} = +.000402 \text{ in.}$$

But $$R_2^{a2} - Y = -.000008 \text{ in.}$$

Hence, $$W - .000394 \text{ in.} = Y$$

Another investigation, the details of which are not given here, gave the equation $W - .000364 \text{ in.} = Y$.

Probable Errors of Observation.

The errors of observation to which the comparisons are subject may be classified as follows: —

(*a.*) Accidental errors of observations for coincidence of micrometer line with defining lines of standard.

(*b.*) Errors due to an imperfect focus of the defining lines under the objective, and which are not included under (*a*).

(*c.*) Errors due to the failure of the thermometer to indicate the real temperature of the standards compared.

The comparisons of T^{a_2} with T^{b_1}, $R_1^{a_2}$ with $R_1^{b_2}$, and $R_1^{a_2}$ with $R_1^{d_2}$, furnish the data for the computation of probable errors of the first and second classes.

Using the formula of Peters we have for the probable error of a *single comparison,* —

$$r = .8453 \frac{[v]}{\sqrt{n(n-1)}}$$

Whence,

From	$T^{a_2} - T^{b_1}$	$r = \pm 0.45\,\mu$
	$R_1^{a_2} - R_1^{b_2}$ (meter)	$r = \pm 0.34\,\mu$
	$R_1^{a_2} - R_1^{b_2}$ (yard)	$r = \pm 0.35\,\mu$

The following include errors of the class (*c*): —

From	$T^{a_2} - C. S.$ (meter)	$r = \pm 0.61\,\mu$
	$T^{a_2} - R_1^{a_2}$ "	$r = \pm 1.44\,\mu$
	$T^{a_2} - R_2^{a_2}$ "	$r = \pm 1.34\,\mu$
	$C. S. - R_1^{a_2}$ (yard)	$r = \pm 0.99\,\mu$
	$C. S. - R_2^{a_2}$ "	$r = \pm 0.49\,\mu$

For the probable error of the final results we have, —

From	$T^{a_2} - T^{b_1}$	$e = \pm 0.07\,\mu$
	$R_1^{a_2} - R_1^{b_2}$ (meter)	$e = \pm 0.06\,\mu$
	$R_1^{a_2} - R_1^{b_2}$ (yard)	$e = \pm 0.06\,\mu$
	$T^{a_2} - C. S.$ (meter)	$e = \pm 0.09\,\mu$
	$T^{a_2} - R_1^{a_2}$ "	$e = \pm 0.22\,\mu$
	$T^{a_2} - R_2^{a_2}$ "	$e = \pm 0.20\,\mu$
	$C. S. - R_1^{a_2}$ (yard)	$e = \pm 0.15\,\mu$
	$C. S. - R_2^{a_2}$ "	$e = \pm 0.07\,\mu$

The only conclusion which can be safely drawn from these results is, that the errors due to temperature are between three and four times as large as the accidental errors of observation. The errors due to imperfect focus may be considered as eliminated in a long series of observations.

The writer regards the values given for the probable errors of the final results as entirely illusory. The fact that the formula for probable errors takes no account of constant errors, needs to be strongly emphasized in this connection. Errors of classes (*b*) and (*c*) are probably of this sort.

Subdivision of Standards.

The subdivisions of the various standards were made with somewhat unequal precision. In the transfers from one standard to another, the errors of subdivision are in a general way reduced; but I have thus far found it impossible to reduce them to zero when the system of corrections applied, includes a change in the entire length.

It hardly needs to be said, that one can measure the subdivisions with much greater accuracy than it is possible to obtain in making the transfers.

The following results have been obtained for the errors of subdivision of the various prototypes discussed in this paper. In bar R_2 the initial defining line is considered to be at the inch end of the bar. In all the other bars the initial line is the middle defining line of the meter and the yard. In the case of the meter the distances are reckoned as follows: —

I = distances reckoned from the middle line *towards* the cm. end.
II = distances reckoned from the middle line *away from* the cm. end.

For the yard,

I = distances reckoned *towards* the inch end.
II = distances reckoned *away from* the inch end.

The columns under the heading Σ contain the corrections reckoned from the initial line. It is to be remembered that a plus sign indicates that the measured space is too short.

SUBDIVISIONS OF $R_2^{a_2}$.

(1 div. = .504 μ.)

FEET.

	July 17.	July 18.	Mean.	Mean of all previous Observations.	Adopted Corrections.		Σ
1	—1.9 div.	—1.3 div.	—1.6 div.	—0.9 div.	—1.2 div.	—0.6 μ	—0.6 μ
2	+6.8 "	+6.4 "	+6.6 "	+4.2 "	+5.4 "	+2.8 μ	+2.2 μ
3	—4.8 "	—5.4 "	—5.1 "	—3.3 "	—4.2 "	—2.2 μ	+0.0 μ

THREE-INCH SPACES OF THE FIRST FOOT.

	July 15.	July 15.	July 17.	July 18.	July 19.	July 20.	July 21.	Means.		Σ
1	—6.6 div.	—5.4 div.	—3.2 div.	—2.8 div.	—3.6 div.	—4.3 div.	—4.1 div.	—4.3 div.	= —2.2 μ	—2.2 μ
2	+2.9 "	+1.0 "	—1.3 "	+0.3 "	—1.0 "	+0.9 "	+0.9 "	+0.2 "	= +0.1 μ	—2.1 μ
3	—3.0 "	—2.3 "	—1.5 "	—3.6 "	—2.1 "	—3.4 "	—2.2 "	—2.6 "	= —1.3 μ	—3.4 μ
4	+6.8 "	+8.6 "	+6.0 "	+6.4 "	+6.7 "	+5.7 "	+5.5 "	+6.8 "	= +3.5 μ	+0.1 μ

INCHES OF THE FIRST THREE-INCH SPACE.

	July 17.	July 18.	July 19.	Mean.			Σ
1	—0.9 div.	—0.5 div.	+0.0 div.	—0.5 div.	=	—0.2 μ	—0.2 μ
2	+1.6 "	+0.8 "	+0.8 "	+1.1 "	=	+0.5 μ	+0.3 μ
3	—0.7 "	—0.3 "	—0.8 "	—0.6 "	=	—0.3 μ	+0.0 μ

For the subdivisions, near the edge of R_2 we have: —

METER.

Halves.

I = —0.7 μ
II = +0.7 μ

Dm. Spaces.

		Σ
I	= —4.4 μ	—4.4 μ
II	= —0.4 μ	—4.8 μ
III	= +1.2 μ	—3.6 μ
IV	= +3.4 μ	—0.2 μ
V	= +0.2 μ	+0.0 μ

Cm. Spaces.

		Σ
I	= —3.4 μ	—3.4 μ
II	= +1.9 μ	—1.5 μ
III	= +0.6 μ	—0.9 μ
IV	= —1.1 μ	—2.0 μ
V	= —0.4 μ	—2.4 μ
VI	= +0.1 μ	—2.3 μ
VII	= —0.6 μ	—2.9 μ
VIII	= +0.0 μ	—2.9 μ
IX	= +0.4 μ	—2.5 μ
X	= +2.6 μ	+0.1 μ

YARD.

Halves.

I = +1.0 μ
II = —1.0 μ

Six-Inch Spaces.

		Σ
I	= +4.0 μ	+4.0 μ
II	= —3.7 μ	+0.3 μ
III	= —0.3 μ	+0.0 μ

Inch Spaces.

		Σ
I	= +0.1 μ	+0.1 μ
II	= +0.2 μ	+0.3 μ
III	= +0.3 μ	+0.6 μ
IV	= —0.4 μ	+0.2 μ
V	= +2.0 μ	+2.2 μ
VI	= —2.2 μ	+0.0 μ

SUBDIVISIONS OF $R_1^{a_2}$.

METER.

Halves.

I = +4.3 μ
II = —4.3 μ

Dm. Spaces.

		Σ
I	= +3.4 μ	+3.4 μ
II	= +0.5 μ	+3.9 μ
III	= —0.3 μ	+3.6 μ
IV	= —2.6 μ	+1.0 μ
V	= —1.0 μ	+0.0 μ

Five Cm. Spaces.

I = —0.1 μ
II = +0.1 μ

YARD.

Halves.

I = +3.8 μ
II = —3.8 μ

Six-Inch Spaces.

		Σ
I	= —2.1 μ	—2.1 μ
II	= +0.0 μ	—2.1 μ
III	= +2.1 μ	+0.0 μ

Inch Spaces.

		Σ
I	= +0.6 μ	+0.6 μ
II	= —0.9 μ	—0.3 μ
III	= +0.0 μ	—0.3 μ
IV	= +0.1 μ	—0.2 μ
V	= +1.2 μ	+1.0 μ
VI	= —1.0 μ	+0.0 μ

Cm. Spaces.

		Σ
I =	−2.2 μ	−2.2 μ
II =	+1.8 μ	−0.4 μ
III =	−1.3 μ	−1.7 μ
IV =	+1.1 μ	−0.6 μ
V =	+0.6 u	+0.0 μ

Subdivisions of G_1.

Meter.

Halves.

	$G_1^{a_2}$	$G_1^{b_1}$
I =	+1.5 μ	+1.2 μ
II =	−1.5 μ	−1.2 μ

Dm. Spaces.

		Σ		Σ
I =	+1.1 μ	+1.1 μ	+0.4 μ	+0.4 μ
II =	−2.3 μ	−1.2 μ	−0.9 μ	−0.5 μ
III =	−0.2 μ	−1.4 μ	+0.3 μ	−0.2 μ
IV =	−0.7 μ	−2.1 μ	+1.4	−1.2 μ
V =	+2.1 μ	+0.0 μ	+1.2 μ	+0.0 μ

Cm. Spaces.

		Σ		Σ
I =	+0.2 μ	+0.2 μ	−1.7 μ	+1.7 μ
II =	+0.4 μ	+0.6 μ	−3.1 μ	−1.4 μ
III =	−0.6 μ	+0.0 μ	+1.4 μ	+0.0 μ
IV =	+1.0 μ	+1.0 μ	+0.3 μ	+0.3 μ
V =	−1.0 μ	+0.0 μ	−0.3 μ	+0.0 μ

Five Cm. Spaces.

I =	−1.1 μ	−3.7 μ
II =	+1.1 μ	+3.7 μ

Yard.

Halves.

	$G_1^{a_2}$	$G_1^{b_1}$
I =	−1.7 μ	−1.8 μ
II =	+1.7 μ	+1.8 μ

Six-Inch Spaces.

		Σ		Σ
I =	−0.2 μ	−0.2 μ	+1.4 μ	+1.4 μ
II =	−0.4 μ	−0.6 μ	−1.9 μ	−0.5 μ
III =	+0.6 μ	+0.0 μ	+0.4 μ	+0.1 μ

Inch Spaces.

		Σ		Σ
I =	−1.3 μ	−1.3 μ	−1.0 μ	−1.0 μ
II =	+1.1 μ	−0.2 μ	−0.7 μ	−1.7 μ
III =	−0.5 μ	−0.7 μ	−0.1 μ	−1.8 μ
IV =	+0.2 μ	−0.5 μ	−2.2 μ	−4.0 μ
V =	+0.5 μ	+0.0 μ	+3.1 μ	−0.9 μ
VI =	+0.0 μ	+0.0 μ	+1.0 μ	+0.1 μ

Subdivisions of G_2.

Meter.

Halves.

	$G_2^{a_2}$	$G_2^{b_2}$	$G_2^{d_2}$
I =	+3.2μ	+1.7μ	−1.1μ
II =	−3.2μ	−1.7μ	+1.1μ

Dm. Spaces.

		Σ		Σ		Σ
I =	+2.2μ	+2.2μ	+1.3μ	+1.3μ	−3.3μ	−3.3μ
II =	−2.5μ	−0.3μ	−1.3μ	+0.0μ	+0.1μ	−3.2μ
III =	−1.2μ	−1.5μ	+1.4μ	+1.4μ	+2.0μ	−1.2μ
IV =	+1.2μ	−0.3μ	−2.8μ	−1.4μ	−0.2μ	−1.4μ
V =	+0.3μ	+0.0μ	+1.5μ	+0.1μ	+1.4μ	+0.0μ

Yard.

Halves.

	$G_2^{a_2}$	$G_2^{b_2}$	$G_2^{d_2}$
I =	+4.0μ	+1.7μ	+1.4μ
II =	−4.0μ	−1.7μ	−1.4μ

Six-Inch Spaces.

		Σ		Σ		Σ
I =	−0.4μ	−0.4μ	+2.6μ	+2.6μ	−1.7μ	−1.7μ
II =	+0.0μ	−0.4μ	−1.3μ	+1.3μ	+0.2μ	−1.5μ
III =	+0.4μ	+0.0μ	−1.3μ	+0.0μ	+1.5μ	+0.0μ

Cm. Spaces.

		Σ		Σ		Σ
I =	−0.1μ	−0.1μ	+1.7μ	+1.7μ	+2.0μ	+2.0μ
II =	+0.2μ	+0.1μ	−2.9μ	−1.2μ	−2.3μ	−0.3μ
III =	−0.1μ	+0.0μ	+0.9μ	−0.3μ	+0.4μ	+0.1μ
IV =	+0.4μ	+0.4μ	−0.2μ	−0.5μ	+0.2μ	+0.3μ
V =	−0.4μ	+0.0μ	+0.5μ	+0 0μ	−0.2μ	+0.1μ

Inch Spaces.

		Σ		Σ		Σ
I =	−1.3μ	−1.3μ	−4.5μ	−4.5μ	−4.2μ	−4.2μ
II =	+1.1μ	−0.2μ	−0.1μ	−4.6μ	−0.2μ	−4.4μ
III =	−0.5μ	−0.7μ	+3.6μ	−1.0μ	+4.0μ	−0.4μ
IV =	+0.3μ	−0.4μ	−1.6μ	−2.6μ	−2.1μ	−2.5μ
V =	+1.3μ	+0.9μ	+3.3μ	+0.7μ	+4.2μ	+1.7μ
VI =	−0.9μ	+0.0μ	−0.7μ	+0.0μ	−1.7μ	+0.0μ

Five Cm. Spaces.

I =	−0.8μ	−3.7μ	−4.8μ
II =	+0.8μ	+3.7μ	+4.8μ

Equation between the Imperial Yard and the Metre des Archives.

The writer presented to the Montreal meeting of the American Association for the Advancement of Science, in 1882, a paper in which the following relation was announced:—

Imperial Yard + 3.37015 inches = Metre des Archives.

I stated at that time, however, with reference to this relation, that for very obvious reasons I should not like to be held to a very strict account with reference to the last decimal figure given, or even to the last two decimal figures.

The problem consists of two parts:—

First, the determination of the relation at 62° F. or 16°.67 C. between the particular yard and meter defined by $R_2{}^{a}{}_2$, and the original standards from which these units were derived.

Second, the measurement of the space 3.370+ inches.

Let M = the true value of the meter $R_2{}^{a_2}$, expressed in terms of the Metre des Archives.

Y' = the true value of the yard $R_2{}^{a_2}$ expressed in terms of the Imperial Yard.

$X = R_2{}^{a_2}$ (meter) — $R_2{}^{a_2}$ (yard).

Then $$X = M - Y'.$$

In the investigation of 1882, a space of four inches was laid off upon a short bar designated B, having the same composition as R_2. This space was subdivided to inches. The third inch was subdivided to tenths of an inch, and the seventh tenth was subdivided to hundredths of an inch. The following relations between the subdivisions of $R_2{}^{a_2}$ and B were then determined.

SUBDIVISIONS OF $R_2^{a_2}$.

Feet.

		Σ
I	$-0.4\,\mu$	$-0.4\,\mu$
II	$+1.6\,\mu$	$+1.2\,\mu$
III	$-1.2\,\mu$	$+0.0\,\mu$

Six-Inch Spaces of First Foot.

I	$= -2.1\,\mu$
II	$= +2.1\,\mu$

Inch Spaces of the First Six Inches.

		Σ
I	$= -0.7\,\mu$	$-0.7\,\mu$
II	$= +1.6\,\mu$	$+0.9\,\mu$
III	$= +0.2\,\mu$	$+1.1\,\mu$
IV	$= -0.5\,\mu$	$+0.6\,\mu$
V	$= -0.7\,\mu$	$-0.1\,\mu$
VI	$= +0.1\,\mu$	$+0.0\,\mu$

Four-Inch Spaces of First Foot.

		Σ
I	$= -0.1\,\mu$	$-0.1\,\mu$
II	$= -2.8\,\mu$	$-2.9\,\mu$
III	$= +2.9\,\mu$	$+0.0\,\mu$

SUBDIVISIONS OF B.

Inches.

		Σ
I	$= +0.5\,\mu$	$+0.5\,\mu$
II	$= +0.1\,\mu$	$+0.6\,\mu$
III	$= -2.2\,\mu$	$-1.6\,\mu$
IV	$= +1.6\,\mu$	$+0.0\,\mu$

Tenths of Third Inch.

		Σ
I	$= +0.3\,\mu$	$+0.3\,\mu$
II	$= +0.2\,\mu$	$+0.5\,\mu$
III	$= +0.9\,\mu$	$+1.4\,\mu$
IV	$= -1.3\,\mu$	$+0.1\,\mu$
V	$= -0.2\,\mu$	$-0.1\,\mu$
VI	$= +0.2\,\mu$	$+0.1\,\mu$
VII	$= -0.4\,\mu$	$-0.3\,\mu$
VIII	$= +0.2\,\mu$	$-0.1\,\mu$
IX	$= -0.3\,\mu$	$-0.4\,\mu$
X	$= +0.4\,\mu$	$+0.0\,\mu$

Hundredths of an Inch.

	Σ
$+0.1\,\mu$	$+0.1\,\mu$
$+0.0\,\mu$	$+0.1\,\mu$
$+0.0\,\mu$	$+0.1\,\mu$
$-0.2\,\mu$	$-0.1\,\mu$
$-0.1\,\mu$	$-0.2\,\mu$
$+0.3\,\mu$	$+0.1\,\mu$
$+0.1\,\mu$	$+0.2\,\mu$
$-0.1\,\mu$	$+0.1\,\mu$
$-0.2\,\mu$	$-0.1\,\mu$
$+0.1\,\mu$	$+0.0\,\mu$

The value of X expressed in the same unit as Y was found to be 3.370319 inches.

Up to this point in the investigation, the following relations had been found:—

$$R_2^{a_2}\text{ (meter)} - 2.6\,\mu = A,$$

$$R_2^{a_2}\text{ (yard)} + 1.6\,\mu = Y,$$

whence

$$1.0000026\ A = [35.999934 + 3.370319]\text{ inches,}$$

and

$$A = 39.37015\text{ inches.}$$

Since the investigation of 1882, it has been possible to make the relation between $R_2^{a_2}$ and the original units more secure through the medium of the yard and meter *C. S.* In the present discussion it was thought better to vary the method of determining the value of X.

Let $x =$ the three-inch space beyond the limits of the yard $R_2^{a_2}$.

$y =$ the distance between the defining line of this space and the defining line at this end of the meter $R_2^{a_2}$.

Then

$$M = Y' + x + y.$$

The value of x was determined by comparing this space with the four three-inch spaces composing the first foot of $R_2^{a_2}$. This was very easily and expeditiously done by setting the stops of the comparator at a distance approximately equal to x, and then comparing each space with this constant distance.

For the measurement of the distance y, a space of two inches was laid off upon a short bronze bar with subdivisions to half-inches. The third half-inch is subdivided into five equal parts, and the third subdivision is again subdivided into ten equal parts. The following relations were found between these subdivisions.

Half-Inch Spaces.

	July 15.	July 17.	July 19.	July 19.	July 20.	July 21.	July 22.	Mean.		Σ
I. =	—4.4div.	—2 1div.	—2.5div.	—2.8div.	—2.8div.	+0.4div	—1.8div.	—2.3div.	= —1.2μ	—1.2μ
II. =	+0.1 "	+1.8 "	+3.5 "	+3.5 "	+2.9 "	+0.5 "	+2.8 "	+2.1 "	= +1.1μ	—0.1μ
III. =	+2.5 "	+2.2 "	—2.1 "	—1.7 "	—1 1 "	—1.3 "	—1.6 "	—0.5 "	= —0.2μ	—0.3μ
IV. =	+1.8 "	—1.9 "	+1.1 "	+1.0 "	+1.0 "	+0.2 "	+0.7 "	+0.6 "	= +0.3μ	+0.0μ

One-Tenth Inch Spaces.

	July 14.	July 15.	July 17.	July 20.	July 21.	Means.		Σ
I. =	+2.9 div.	+1.5 div.	+2.3 div.	+2.1 div.	+1.7 div.	+2.1 div.	= +1.1 μ	+1.1 μ
II. =	+1.8 "	+0.6 "	—0.4 "	+0.6 "	+0.1 "	+0.5 "	= +0.2 μ	+1.3 μ
III. =	—0.9 "	—1.3 "	+0.2 "	—1.0 "	—1.0 "	—0.8 "	= —0.4 μ	+0.9 μ
IV. =	—1.0 "	+0.7 "	—0.9 "	—1.1 "	+0.7 "	—0.3 "	= —0.1 μ	+0.8 μ
V. =	—2.8 "	—1.6 "	—1.0 "	—0.5 "	—1.5 "	—1.5 "	= —0.8 μ	+0.0 μ

One-Hundredth Inch Spaces.

	July 14.	July 15.	July 17.	July 21.	Means.		Σ
I. =	—1.7 div.	— 1.3 div.	+0.5 div.	+1.2 div.	—0.3 div.	= —0.2 μ	—0.2 μ
II. =	—3.1 "	— 6.4 "	—4 5 "	—5.9 "	—4.5 "	= —2.3 μ	—2.5 μ
III. =	+3.7 "	+ 3 6 "	+4.6 "	+4.3 "	+4.0 "	= +2.0 μ	—0.5 μ
IV. =	+0.9 "	+ 1.1 "	+0.9 "	—0.7 "	+0.5 "	= +0.3 μ	—0.2 μ
V. =	—1.4 "	— 0.3 "	—0.5 "	+0.4 "	—0.5 "	= —0.2 μ	—0.4 μ
VI. =	—8.0 "	— 8.2 "	—9.5 "	—7.9 "	—8.4 "	= —4.3 μ	—4.7 μ
VII. =	+8.5 "	+11.4 "	+8.6 "	+8.7 "	+9.3 "	= +4.8 μ	+0.1 μ
VIII. =	+0.6 "	+ 2.5 "	+1.3 "	+1.0 "	+1.3 "	= +0.6 μ	+0.7 μ
IX. =	+0.3 "	+ 0.5 "	—2.4 "	—1.4 "	—0.8 "	= —0.4 μ	+0.3 μ
X. =	+0.2 "	— 3.0 "	+1.0 "	+0.0 "	—0.5 "	= —0.3 μ	+0.0 μ

Comparison of 2 *Inches of Scale B with the first* 2 *Inches of* $R_2^{a_2}$.

From observations of July 21,

$$B + 1.9 \text{ div.} = \text{first 2 inches of } R_2{}^{a}{}_{2}.$$

From observations of July 22,

$$B + 1.8 \text{ div.} = \text{first 2 inches of } R_2^{a_2}.$$

The following relations for x and y were found, x being expressed in terms of the first foot of $R_2^{a_2}$ divided by 4, and y in terms of line 37 of the scale B.

	x	y
July 15	3 inches +13.2 div.	+2.9 div.
" 15	" +19.1 "	+2.8 "
" 17	" +17.4 "	+5.0 "
" 18	" +16.9 "	+4.0 "
" 19	" +16.3 "	+2.2 "
" 20	" +14.3 "	+2.6 "
" 21	" +16.2 "	+4.5 "
" 22	" +16.3 "	+2.1 "
Means	3 inches +16.3 div. = 3.000321 inches.	+3.3 div. = .000065 inch.

Expressed in terms of Y, x becomes	3.000327 inches.
and $x + y$ expressed in the same unit becomes	3.000392 "

This quantity is now to be corrected by the amount of the error of line 37 of the third half-inch of the scale B. We have,

$$2 \text{ inches of } B + [1.9 + 0.6 - 0.2] \text{ div.} = \frac{1}{18} Y.$$

The correction to line 37 of the third inch of scale B is, therefore,

$$2 \text{ inches} - \big[1.7 + [1.8 - 1.0 + 0.2]\big] \text{ div.} = \frac{1}{72} Y.$$

Expressed in terms of Y, we have, finally,

$$x + y = 3.370339 \text{ inches.}$$

From the observations of 1882,

$$x + y = 3.370319 \text{ inches.}$$

Adopting the mean between these values, we have

$$M + x + y = [36.999992 + 3.370329] \text{ inches.}$$

We have therefore, finally,

$$1.0000013\, A = 39.370321 \text{ inches,}$$

and

$$A = \mathbf{39.37027\ inches.}$$